Model error and covariance estimation in data assimilation

Alberto Carrassi
acarrassi@ic3.cat

Climate Foreasting Unit - **CFU**
Catalan Institute for Climate Science - **IC3**
Spain

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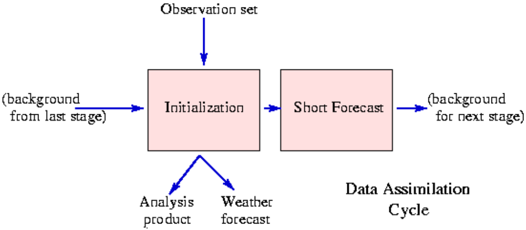
Exploring the Use of Data Assimilation for the Detection and Attribution of Climate Change

Data Assimilation is the entire sequence of operations that, starting from the observations and possibly from a statistical/dynamical knowledge about a system, provides an estimate of its state

The main fields of applications in geophysics are:

- initialize weather prediction
- produce reanalysis
- parameter estimation



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- observations (synoptic profiles, onboard measurements, remote sensing, etc...)
- background field (climatological, short range forecast)
- evolution dynamics (set of differential equations, numerical model ...)

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But ...

- realistic models are nonlinear and chaotic so that errors amplify rapidly and are subject to a flow-dependent dynamics - an accurate flow-dependent description of the forecast error is crucial
- models are not perfect - incorrect parametrizations of physical processes, numerical discretizations, unresolved scales, etc..

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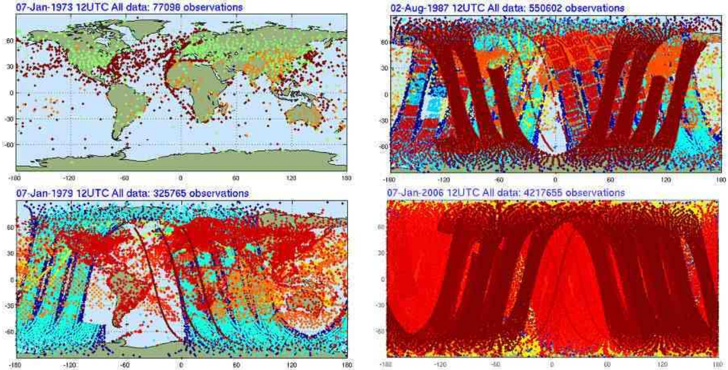
In the last decades the accuracy of initial conditions has improved:

- observational network has been enlarged and refined (a major contribution came from remote sensing measurements)
- there has been a flourishing of data assimilation techniques aimed at a flow dependent description of the forecast error (KF-like algorithms, Monte Carlo and Deterministic filters, AUS, 4DVar, ...)



controlling errors: **what about model error ?**

In the past, model error has been considered small with respect to the (growth of) initial condition error, and thus often neglected

Nowadays model error is recognized as a main source of uncertainty in NWP, seasonal and climate prediction

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- lack of a general framework for model error dynamics

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OBJECTIVES

1. Identifying some general laws for the evolution of the model error dynamics (with suitable *application-oriented* approximations)
2. Use of these dynamical laws to prescribe the model error statistics required by DA algorithms

The posing of the problem

Let assume to have the model:

$$\frac{d\mathbf{x}(t)}{dt} = f(\mathbf{x}, \lambda)$$

used to describe the true process:

$$\frac{d\hat{\mathbf{x}}(t)}{dt} = \hat{f}(\hat{\mathbf{x}}, \hat{\mathbf{y}}, \lambda^{'}) + \epsilon\hat{g}(\hat{\mathbf{x}}, \hat{\mathbf{y}}, \lambda^{'})$$
$$\frac{d\hat{\mathbf{y}}(t)}{dt} = \hat{h}(\hat{\mathbf{x}}, \hat{\mathbf{y}}, \lambda^{'})$$

- $\hat{g}(\hat{\mathbf{x}}, \hat{\mathbf{y}}, \lambda^{'})$ represents the dynamics associated to extra processes not accounted for by the model;
- $\hat{h}(\hat{\mathbf{x}}, \hat{\mathbf{y}}, \lambda^{'})$ - unresolved scale

CASE I - Parametric Error

- the model resolves all the relevant scales $\Rightarrow \hat{h} = 0$ and $f = \hat{f}$
- error in the parameter $\delta\lambda \neq 0$
- set $\epsilon = \gamma\delta\lambda$

Estimation error evolution in the linear approximation

$$\delta\mathbf{x}(t) \approx \mathbf{M}_{t,t_0}\delta\mathbf{x}_0 + \int_{t_0}^{t} d\tau \mathbf{M}_{t,\tau}\delta\mu(\tau) = \delta\mathbf{x}^{ic}(t) + \delta\mathbf{x}^{m}(t)$$

where

$$\delta\mu = [\frac{\partial\mathbf{f}}{\partial\lambda}|_{\lambda} + \gamma g(\mathbf{y}(t), \lambda^{'})]\delta\lambda$$

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- The model error acts as a deterministic process
- The important factor controlling the evolution is $\delta\mu(t)$
- In view of the presence of the propagator M, the flow instabilities are expected to influence the model error dynamics

Model error covariance and correlation - CASE I

Model error covariance

$$\mathbf{P}^m(t) = \int_{t_0}^{t} d\tau \int_{t_0}^{t} d\tau^{'} \mathbf{M}_{t,\tau} < (\delta\mu(\tau))(\delta\mu(\tau^{'}))^T) > \mathbf{M}^T_{t,\tau^{'}}$$

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Model error correlation

$$\mathbf{P}^m(t_1, t_2) = \int_{t_0}^{t_1} d\tau \int_{t_0}^{t_2} d\tau^{'} \mathbf{M}_{t_1,\tau} < \delta\mu(\tau)\delta\mu(\tau^{'})^T > \mathbf{M}_{t_2,\tau^{'}}^T$$

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These covariance and correlations are exactly what we need in DA !

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These covariance and correlations are exactly what we need in DA !

These equations are NOT suitable for realistic geophysical applications - Some approximation is required

Short time approximation - CASE I

Model error covariance

$$\mathbf{P}^m(t) \approx < \delta\mu_0 \delta\mu_0^T > (t - t_0)^2$$

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- The covariance $\mathbf{Q}$ embeds the information on the model error through $\delta\lambda$ and the functional dependence of the dynamics on the parameters.

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- The main factor determining this evolution is the covariance of $\delta\mu$ at $t = t_0$, $\mathbf{Q} = < \delta\mu_0 \delta\mu_0^T >$.
- The covariance $\mathbf{Q}$ embeds the information on the model error through $\delta\lambda$ and the functional dependence of the dynamics on the parameters.
- Once $\mathbf{Q}$ is known, $\mathbf{P}^m$ can be computed at any time within the short time regime

CASE II - Error due to unresolved scales

- the model does not describe the scale given by $\hat{h}(\hat{\mathbf{x}}, \hat{\mathbf{y}}, \lambda^{'})$
- assume correct parameter, $\delta\lambda = 0$, and set $\epsilon = 0$

Estimation error evolution in the resolved scale

$$\delta\mathbf{x}(t) = \mathbf{x}(t) - \hat{\mathbf{x}}(t) = \delta\mathbf{x}_0 + \int_{t_0}^{t} d\tau(f(\mathbf{x}, \lambda) - \hat{f}(\hat{\mathbf{x}}, \hat{\mathbf{y}}, \lambda))$$

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Evolution of the estimation error covariance in the resolved scale

$$\mathbf{P}(t) =< \delta\mathbf{x}_0 \delta\mathbf{x}_0^T > + \int_{t_0}^{t} d\tau \int_{t_0}^{t} d\tau^{'} < [f(\mathbf{x}, \lambda) - \hat{f}(\hat{\mathbf{x}}, \hat{\mathbf{y}}, \lambda)][f(\mathbf{x}, \lambda) - \hat{f}(\hat{\mathbf{x}}, \hat{\mathbf{y}}, \lambda)]^T >$$

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- the correlation between i.c. and model error neglected (standard hyp. in DA)
- the important factor controlling the evolution is the difference between the velocity fields $f(\mathbf{x}, \lambda) - \hat{f}(\hat{\mathbf{x}}, \hat{\mathbf{y}}, \lambda)$

Short Time Approximation - CASE II

- the contribution $f(\mathbf{x},\lambda) - \hat{f}(\hat{\mathbf{x}},\hat{\mathbf{y}},\lambda)$ is treated as a deterministic process
- the short time evolution of $\mathbf{P}(t)$ reads:

$$\mathbf{P}(t) \approx < \delta\mathbf{x}_0\delta\mathbf{x}_0^T > + < [f(\mathbf{x},\lambda) - \hat{f}(\hat{\mathbf{x}},\hat{\mathbf{y}},\lambda)][f(\mathbf{x},\lambda) - \hat{f}(\hat{\mathbf{x}},\hat{\mathbf{y}},\lambda)]^T > t^2 + O(3)$$

DA in the presence of model error

Can we incorporate the short-time approximation for the model error covariance in the context of DA procedures ?

Specific goals:

1. **Computation of the model error covariance in the sequential data assimilation - EKF**
2. **Computation of the model error correlations in the weak-constraint 4DVar**

Extended Kalman Filter (EKF) in the presence of model error

$$\boxed{\text{Input } \mathbf{x}^a,\ \mathbf{P}^a}$$

$$\boxed{\text{Forecast: } \mathbf{x}^f = \mathcal{M}(\mathbf{x}^a)\ \&\ \mathbf{P}^f = \mathbf{M}\mathbf{P}^a\mathbf{M}^T + \mathbf{P}^m}$$

$$\boxed{\text{Analysis Update: } \mathbf{x}^a = [\mathbf{I} - \mathbf{K}\mathbf{H}]\,\mathbf{x}^f + \mathbf{K}\mathbf{y}^o\ \&\ \mathbf{P}^a = [\mathbf{I} - \mathbf{K}\mathbf{H}]\mathbf{P}^f}$$

$\mathbf{P}^m$ - Model Error Covariance Matrix

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$\mathbf{P}^m$ - Model Error Covariance Matrix

Estimate $\mathbf{P}^m$ using the short time approximation

Extended Kalman Filter in the presence of parametric error

CASE I - Parametric Error

$$\mathbf{P}^m \approx < \delta\mu_0 \delta\mu_0^T > \tau^2 = \mathbf{Q}\tau^2$$

...needs to estimate $\mathbf{Q}$

Extended Kalman Filter in the presence of parametric error

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...needs to estimate **Q**

Two solutions proposed:

1. Statistically based on a priori information – Short Time EKF (**ST-EKF**)

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...needs to estimate $\mathbf{Q}$

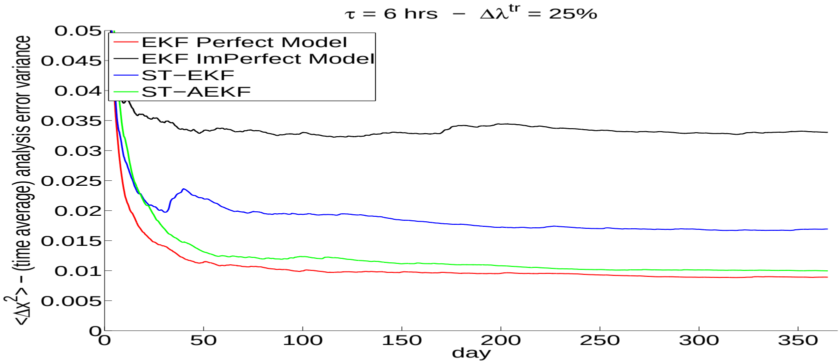
Two solutions proposed:

1. Statistically based on a priori information – Short Time EKF (**ST-EKF**)
2. Dynamically (on the fly) using a state/parameter estimation approach – Short Time Augmented EKF (**ST-AEKF**)

Parametric Error - Numerical Analysis with ST-EKF and ST-AEKF

Carrassi, Vannitsem, Nicolis (2008) QJRMS and Carrassi & Vannitsem (2011) QJRMS

- Prototype of nonlinear chaotic dynamics (Lorenz, 1996): $\frac{dx_i}{dt} = \alpha(x_{i+1} - x_{i-2})x_{i-1} - \beta x_i + F \qquad 1 \leq i \leq 36$
- ST-EKF - **Q** estimate statistically and then kept fixed along the assimilation cycle
- ST-AEKF - **Q** estimated online by measuring system's observables - State Augmented formulation
  - augmented system $\mathbf{z} = (\mathcal{M}(\mathbf{x}), \mathcal{F}(\lambda))^T$
  - at analysis time the state and parameters are estimated along with their associated uncertainty (covariances) and cross correlations
  - the updated parametric error covariance, $\mathbf{P}^a_\lambda = < \delta\lambda\delta\lambda^T >$, is then used to update $\mathbf{Q} \Rightarrow \mathbf{P}^m$



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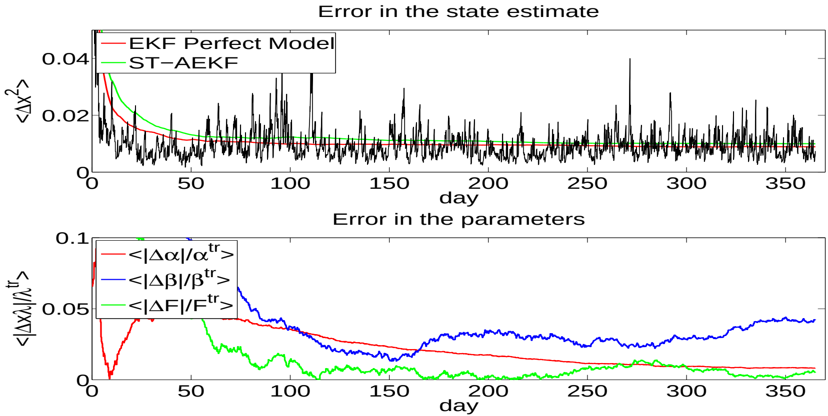
Carrassi, Vannitsem, Nicolis (2008) QJRMS and Carrassi & Vannitsem (2011) QJRMS

Q estimated online by measuring system's observables - State Augmented formulation

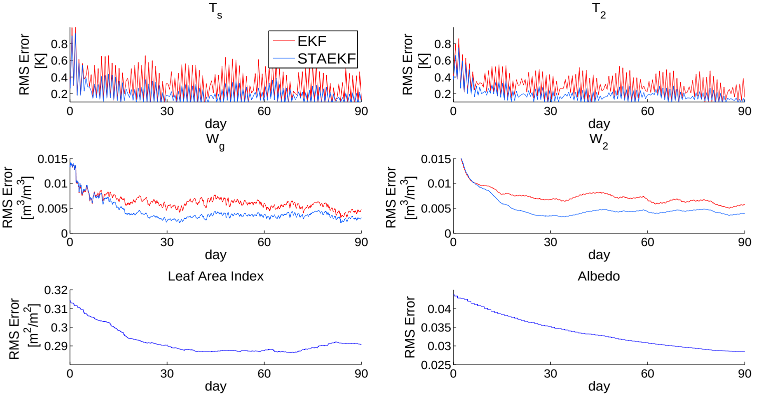
- simultaneous estimate of the three parameters
- results averaged over an ensemble (O(100)) of experiments



Parametric Error - Numerical Analysis with ST-EKF and ST-AEKF

Carrassi, Hamdi, Vannitsem, Termonia (2012) ASL

- Land Surface model ISBA (Mahfouf and Noilhan, 1996)
- State Variables: soil temperature (T_s and T_2) and moisture content (w_g and w_2).
- Observations of screen-level variables (temperature and humidity at 2 meter)
- Parametric error in the *Leaf Area Index* (LAI) and *Albedo*
- Comparison between EKF and ST-AEKF



Error due to unresolved scales – ST-EKF

CASE II - Error of Unresolved Scales $\Rightarrow \mathbf{P}^m \approx < (f-\hat{f})(f-\hat{f})^T > \tau^2$

...needs to estimate the statistics of the vel. fields discrepancy.

Solution proposed:

- Use of the analysis increments of a reanalysis data-set :

$$f-\hat{f} = \frac{d\mathbf{x}}{dt} - \frac{d\hat{\mathbf{x}}}{dt} \approx \frac{\mathbf{x}_r^f(t+\tau_r) - \mathbf{x}_r^a(t)}{\tau_r} - \frac{\mathbf{x}_r^a(t+\tau_r) - \mathbf{x}_r^a(t)}{\tau_r} = \frac{\delta \mathbf{x}_r^a}{\tau_r} \Rightarrow$$

$$\mathbf{P}^m(t) \approx < \delta \mathbf{x}_r^a \delta {\mathbf{x}_r^a}^T > \frac{\tau^2}{\tau_r^2}$$

- τ_r reanalysis assimilation interval
- τ current assimilation interval

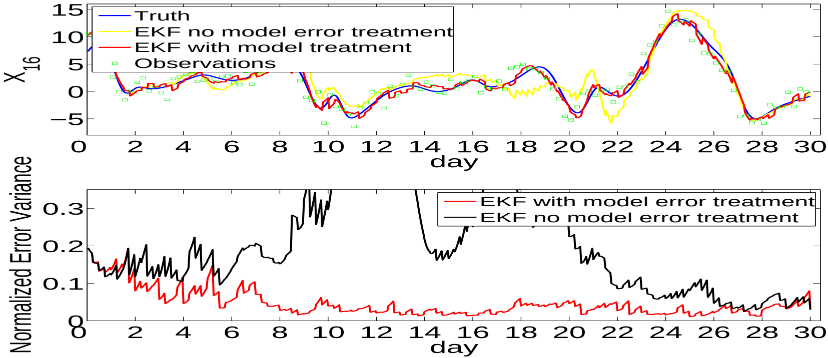
Error due to unresolved scales – ST-EKF

Carrassi & Vannitsem (2011) IJBC

- Lorenz (1996) with two scales (*large scale* - **x**; *small scale* - **y**)
- 12 regular observations of the large scale, **x**, only

$$\frac{dx_i}{dt} = (x_{i+1} - x_{i-2})x_{i-1} - x_i + F - \frac{hc}{b}\sum_{j=1}^{10} y_{j,i}, \qquad i = \{1, \ldots, 36\}$$

$$\frac{dy_{j,i}}{dt} = -cby_{j+1,i}(y_{j+2,i} - y_{j-1,i}) - cy_{j,i} + \frac{hc}{b}x_i, \qquad j = \{1, \ldots, 10\}$$



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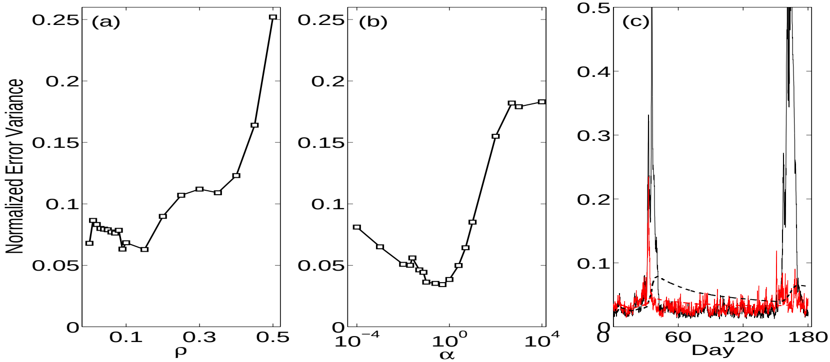
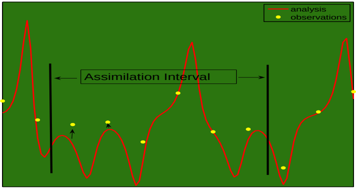
Comparison with the EKF employing the inflation of the $\mathbf{P}^f$ as a tool to account for model error

- (a) - EKF; Inflation procedure on the $\mathbf{P}^f \rightarrow (1+\rho)\mathbf{P}^f$
- (b) - ST-EKF; Tuning of $\mathbf{P}^m \rightarrow \alpha\mathbf{P}^m$ ($\mathbf{P}^m$ estimated statistically and then kept fixed)
- (c) - Analysis Error Comparison ST-EKF ($\alpha = 0.5$ red line) and EKF ($\rho = 0.09$ black line)



4DVar in the presence of model error - Short Time Weak Constraint 4DVar



- assimilate observations distributed over the time window τ

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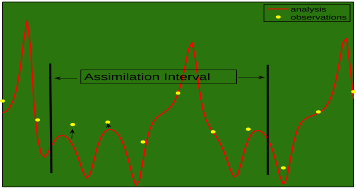

- assimilate observations distributed over the time window τ
- analysis state as the minimum of a cost-function:

$$2J = \int_0^\tau \int_0^\tau (\delta \mathbf{x}_{t_1}^m)^T (\mathbf{P}^m)^{-1}_{t_1 t_2} (\delta \mathbf{x}_{t_2}^m) dt_1 dt_2 + \sum_{k=1}^{M} \epsilon_{\mathbf{k}}{}^T \mathbf{R}_k^{-1} \epsilon_{\mathbf{k}} + \epsilon_b^T \mathbf{B}^{-1} \epsilon_b$$

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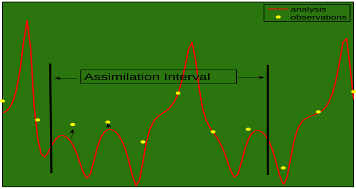

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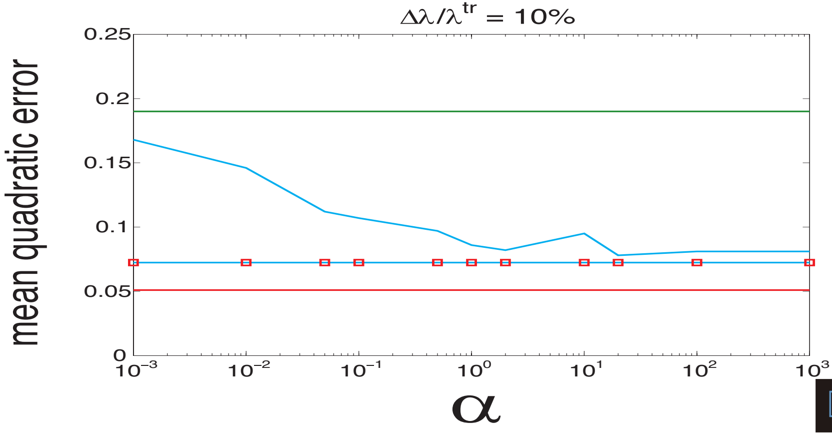
Estimate model error covariances/correlations using $\mathbf{P}(t_1, t_2) \approx \mathbf{Q}(t_1 - t_0)(t_2 - t_0)$

Results weak-constraint 4DVar

Carrassi and Vannitsem, 2010 (MWR)

- Lorenz 3-variable (1963) system
- Assimilation interval $\tau = 8$ time-steps, Obs frequency $\Delta t_{obs} = 2$ time-steps
- **Strong-constraint** - **Short-time weak constraint 4DVar** - **Weak constraint 4DVar with uncorrelated model error: with $P^m_t = \alpha \mathbf{B}$ (blue) or $P^m_t = \mathbf{Q}(t - t_0)^2$ (blue with red marks)**



Conclusions and Perspectives

- the proposed formulations gave encouraging results in the framework of both sequential and variational assimilation
- treating the model error as a deterministic process makes possible to derive short-time approximations for the error covariance suitable for DA applications
- the estimation of model error covariances is based on fundamental features rather than estimated using ad-hoc procedures
- the model error statistics are easily adaptable to different observational frequencies and/or assimilation intervals

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thanks