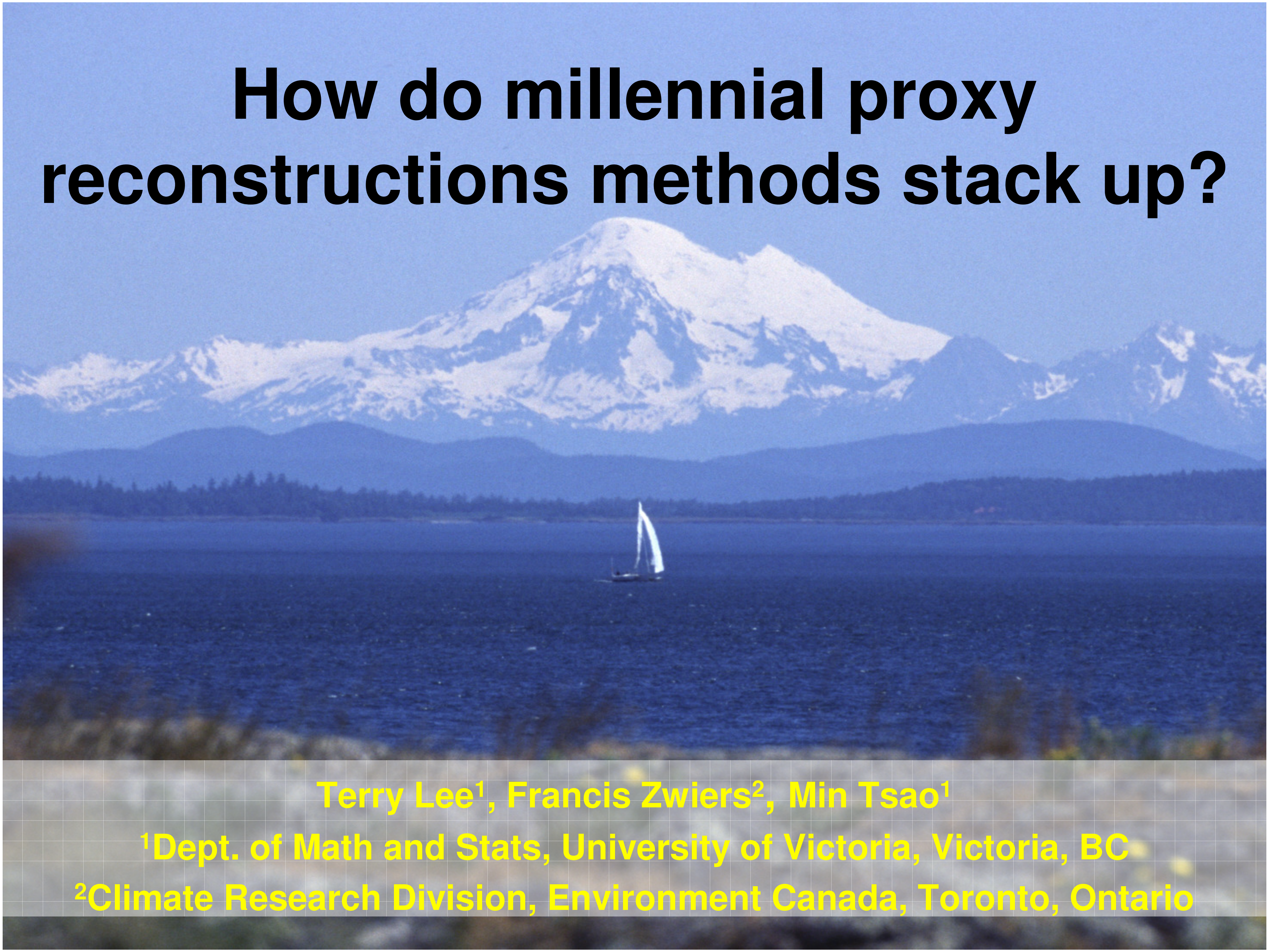
How do millennial proxy reconstructions methods stack up?



Terry Lee[1], Francis Zwiers[2], Min Tsao[1]

[1]Dept. of Math and Stats, University of Victoria, Victoria, BC

[2]Climate Research Division, Environment Canada, Toronto, Ontario

$$\frac{dX}{dt} = -\sigma X + \sigma Y + f_0 \cos\theta$$
$$\frac{dY}{dt} = -XZ + rX - y + f_0 \sin\theta$$
$$\frac{dZ}{dt} = xy - bZ,$$

σ= 10, r=28, b=8/3

Allen and Stott, 2003, Climate Dynamics

Section 4

Distribution of1-day averages

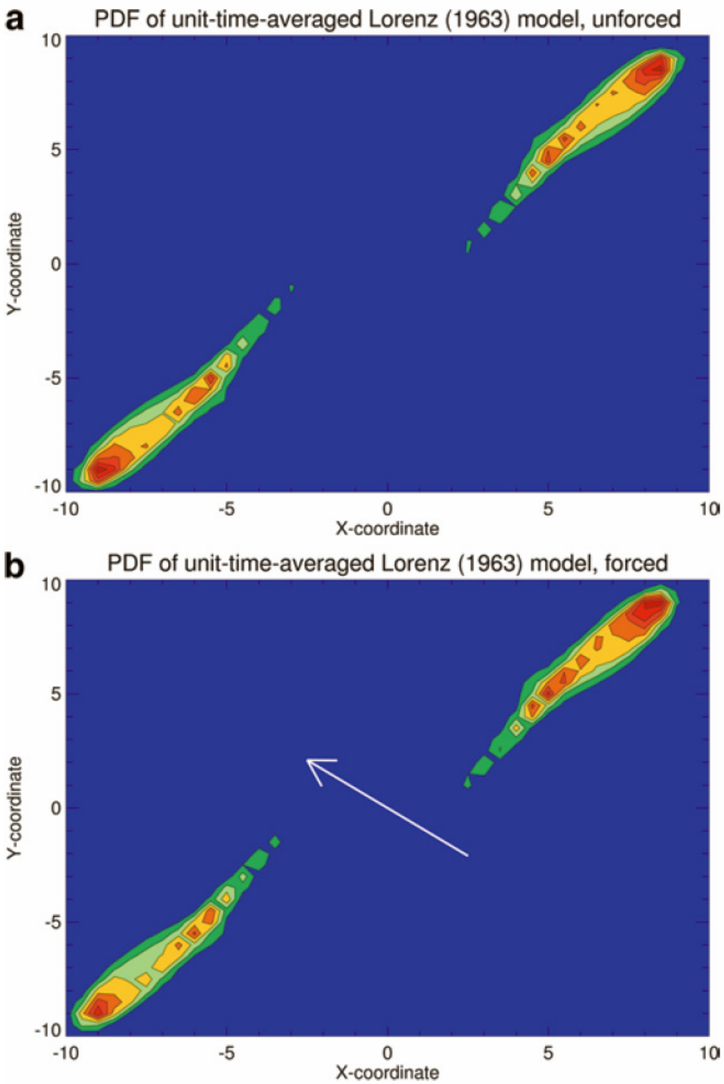


Fig. 3 **a**: estimate of the attractor probability density function (PDF) of the unforced Lorenz (1963), system. The plot shows a two-dimensional histogram of the location of "one-Lorenz-day" time-averaged values of the (X, Y) variables obtained from a long integration. **b**: PDF after imposing a steady forcing in the (X, Y) plane in the direction shown by the *arrow*, following Palmer (1999)

Distribution of500-day averages

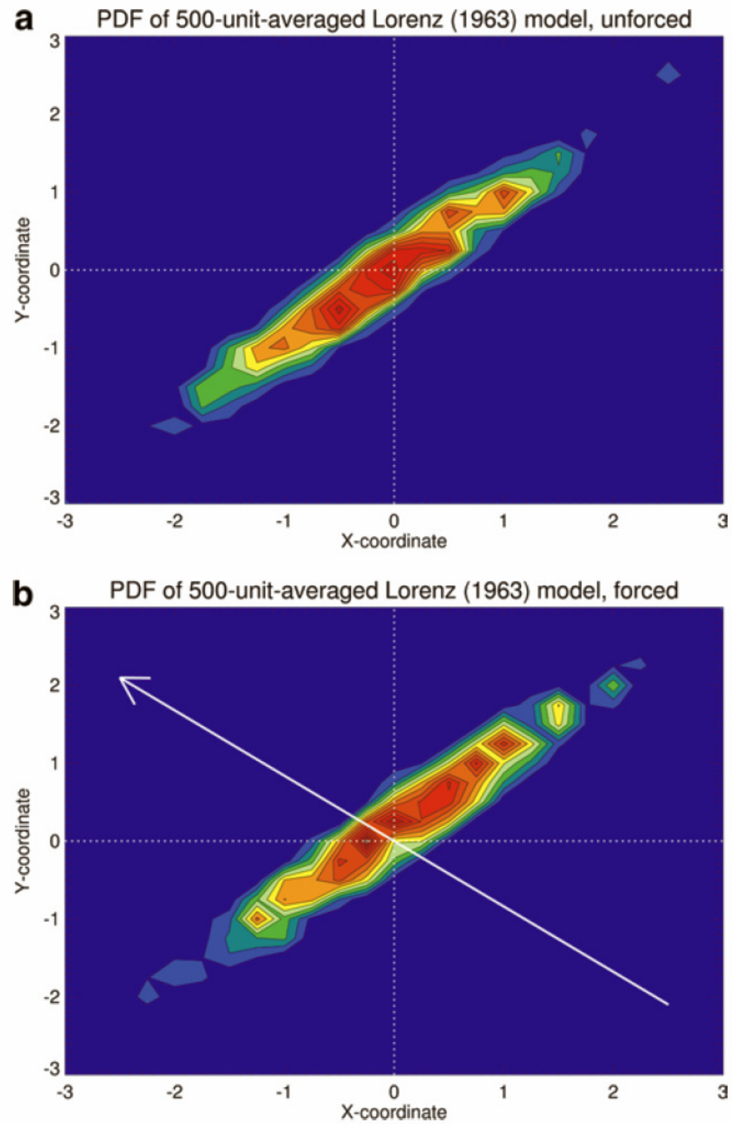


Fig. 4a, b As Fig. 3, but based on 500-Lorenz-day averaged data, to show the impact of time-averaging on the distributional properties of variability generated by a chaotic system: a simple consequence of the Central Limit Theorem

PACIFIC CLIMATE IMPACTS CONSORTIUM

University of Victoria

Outline

- Background
- Reconstruction calibration techniques
- Their evaluation
- Conclusions
- Two extensions
- Comments??

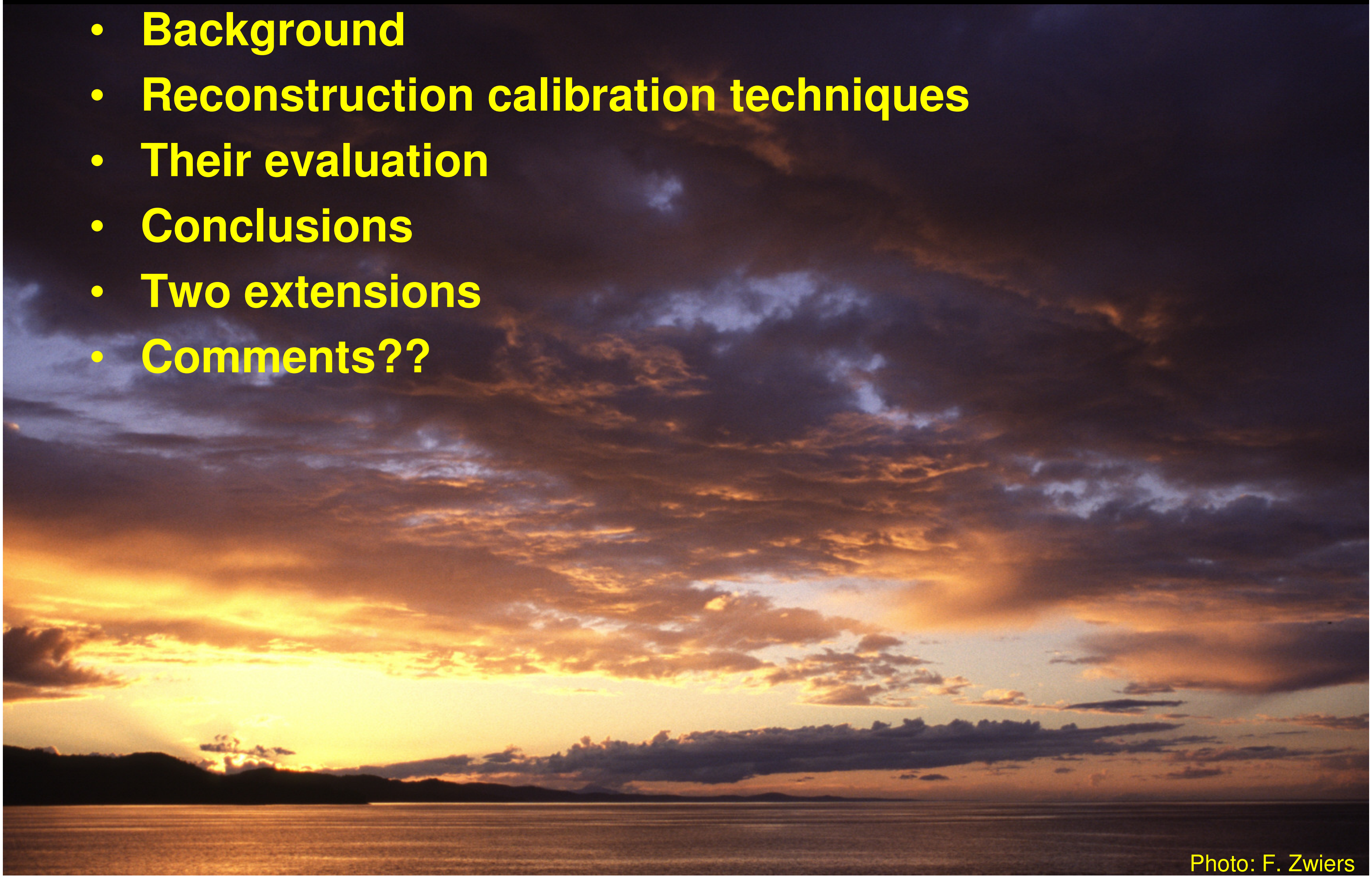
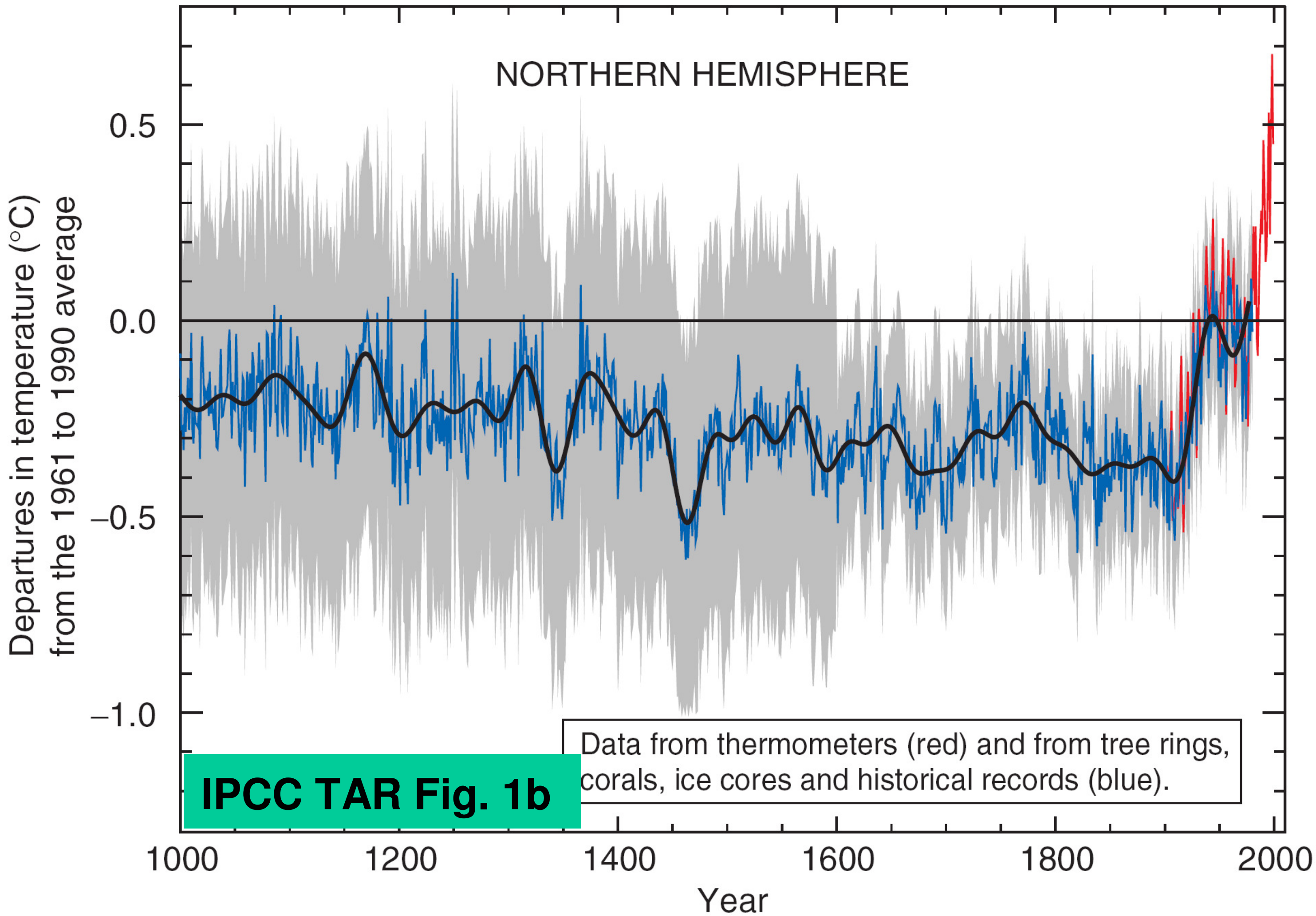


Photo: F. Zwiers

The “hockey stick” (TAR SPM)



- “It is ***likely*** that the 1990s have been the warmest decade and 1998 the warmest year of the millennium”

PACIFIC CLIMATE IMPACTS CONSORTIUM

University of Victoria

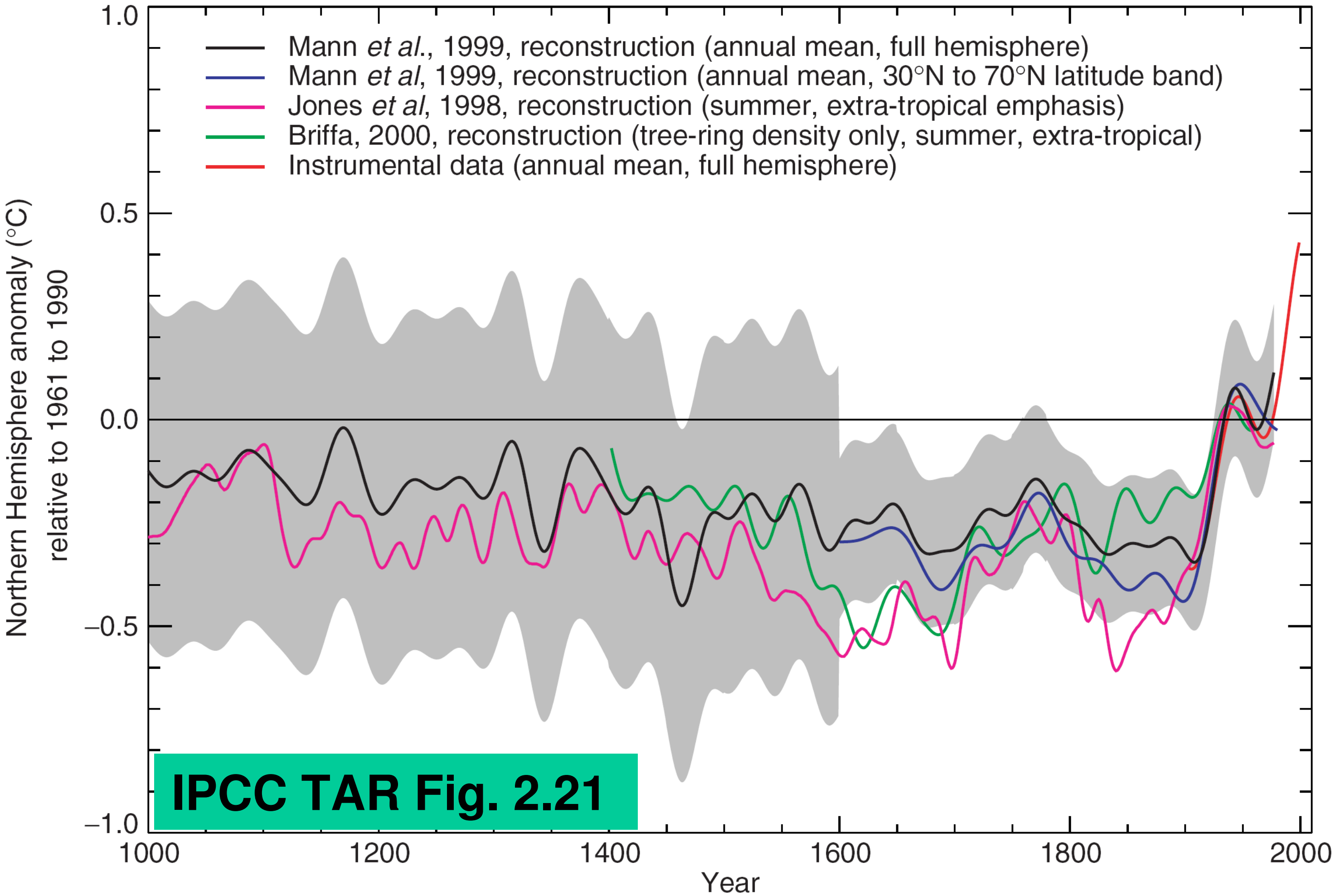
The “hockey-stick” debate …

- Iconic figure and bold statement drew lots of interest from the sceptics
- Some Canadians particularly vocal
- Criticism focused on
  - Statistical method (influence of centering on EOFs)
  - Choice of proxies (use of Bristlecone Pines)

PACIFIC CLIMATE IMPACTS CONSORTIUM

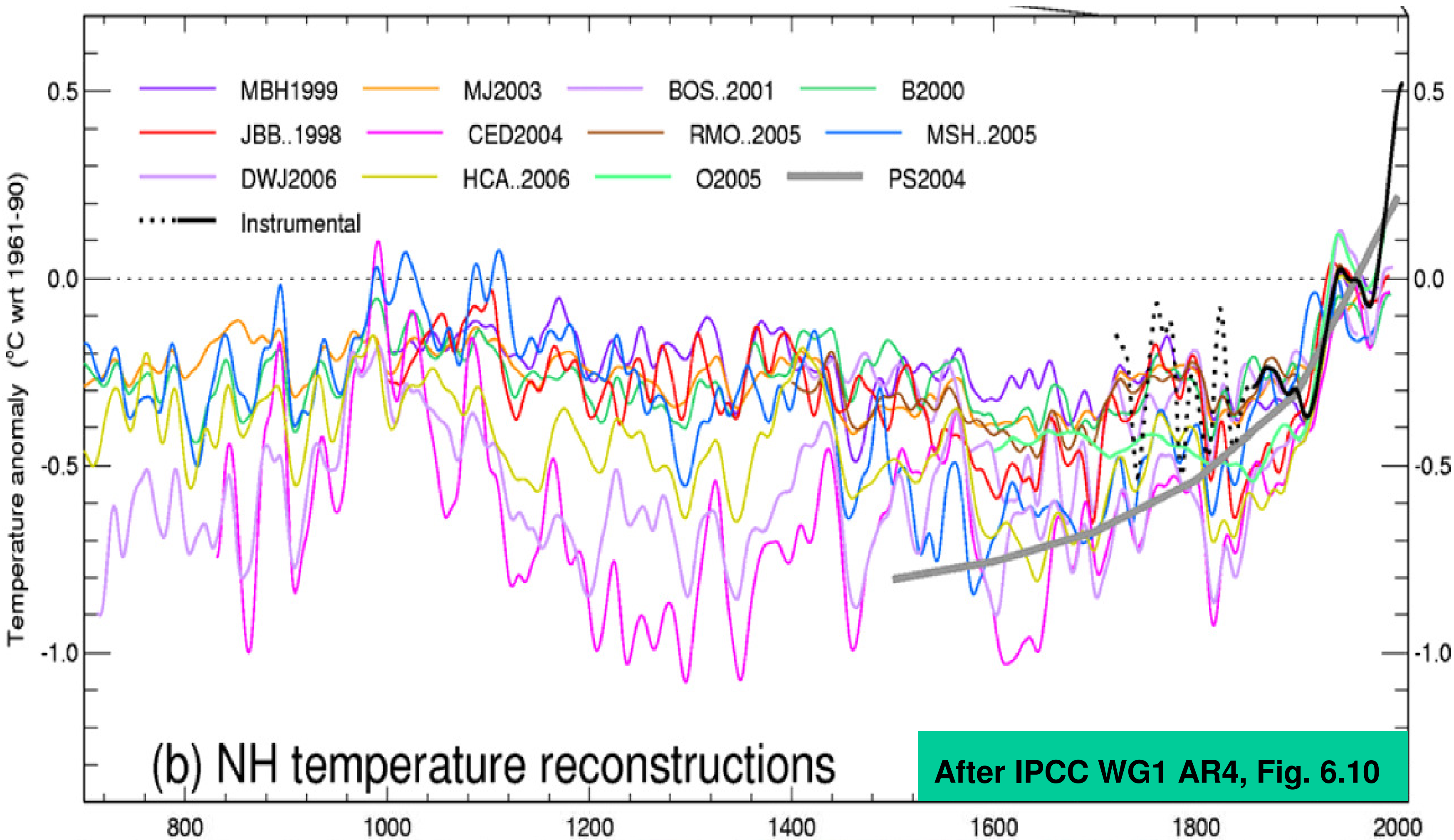
University of Victoria

There was another figure in the TAR …



- Indicating that the general form was robust,
- and that some reconstructions were more variable than others

Reconstructions at time of AR4



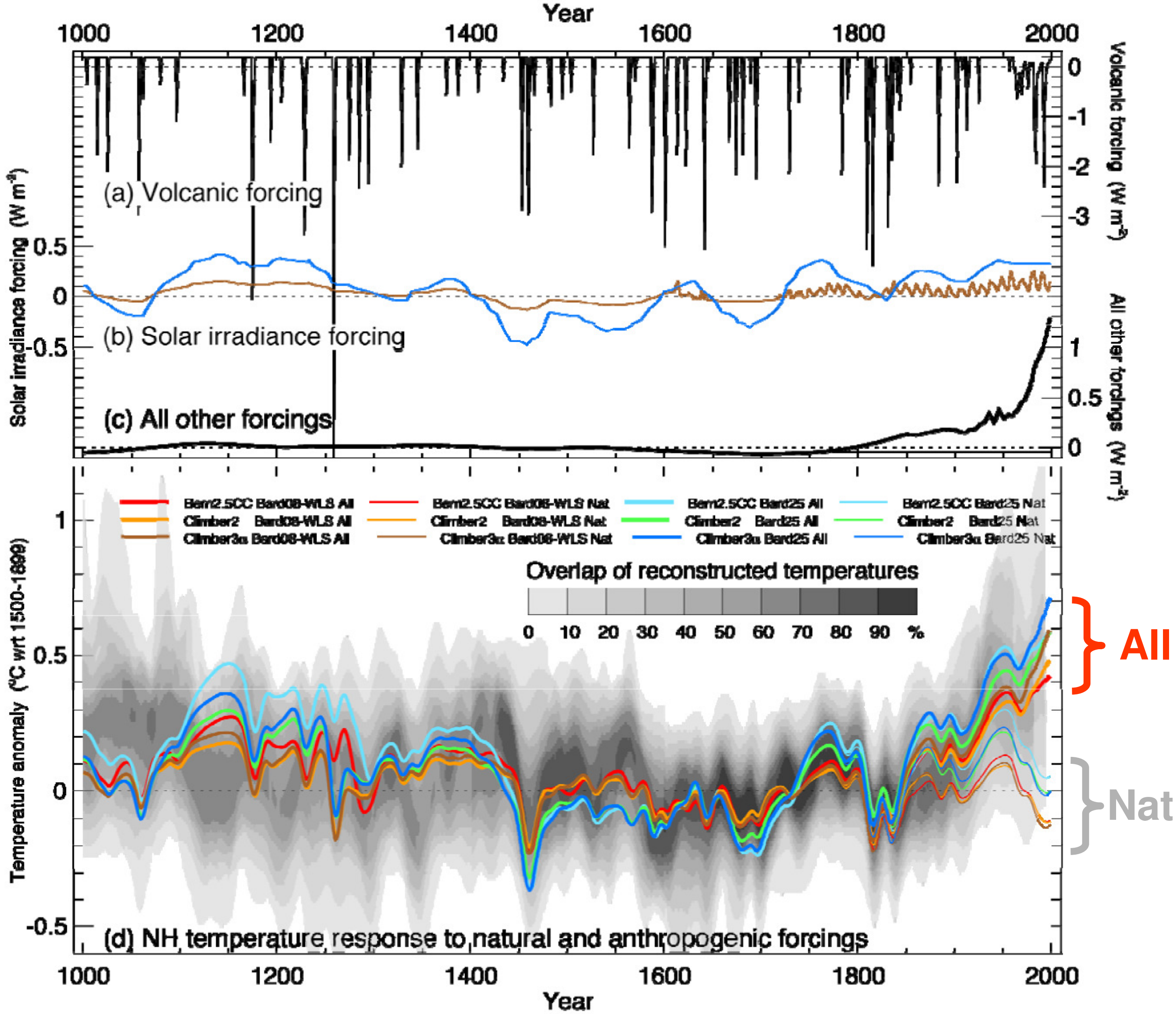
PACIFIC CLIMATE IMPACTS CONSORTIUM

University of Victoria

Interpreting the reconstructions with models

- There now exist several long climate simulations with reconstructed volcanic, solar and GHG forcing
  - With AOGCMs (ECHO-G, CSM 1.4, HadCM3 …)
  - EMICS
  - EBMS
  - (Note – now even more as a consequence of CMIP5)
- These seem able to reproduce some of the features seen in the reconstructions…
- Detection studies confirm the presence of natural and anthropogenic signals…

Temperature of last millenium



After IPCC WG1 AR4, Fig. 6.14

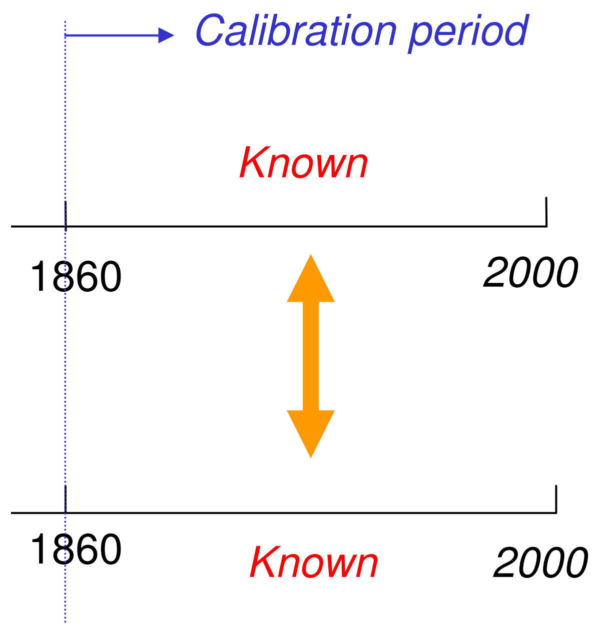
IPCC AR4 Assessment

- Average Northern Hemisphere temperatures during the second half of the 20th century were ***very likely*** higher than during any other 50-year period in the last 500 years and ***likely*** the highest in at least the past 1300 years.
- Assessment refers to longer time scales than in the TAR
- Many more reconstructions available
- Some detection results indicating external influence, detectable in all reconstructions that were examined
- Some inter-comparison of reconstruction techniques
- We have now undertaken a more complete assessment of available techniques (although still far from exhaustive)

The reconstruction techniques

The Reconstruction Problem

Training period



Identify a statistical relationship between a collection proxies and NH temperature

Two types of reconstruction techniques

- **CPS – composite plus scale**
  - Average (or composite) proxies into some index (e.g., just average, and make dimensionless)
  - Calibrate the composite to hemispheric mean temperature from instrumental data
- **CFR – climate field reconstruction**
  - EOF regression, or other technique, to reconstruct hemispheric temperature field
    - Used, for example, to reconstruct SSTs back into 1800's using sparse instrumental data
  - Spatially average the reconstructed field to estimate hemispheric mean temperature

Reconstruction techniques

CPS

- Ordinary Least Squares: $T_t = \beta P_t + \varepsilon_t$
- Total Least Squares: $T_t = \beta(P_t - \eta_t) + \varepsilon_t$
- Variance Matching: $T_t = \beta P_t + \varepsilon_t$, $\hat{\beta} = [V(T)/V(P)]^{1/2}$
- Inverse Regression: $P_t = \beta T_t + \varepsilon_t$
- Kalman Filter/Smoother: $P_t = \beta T_t + \varepsilon_t$, $T_t = \varphi T_{t-1} + \delta F_t + \omega_t$

CFR

- MBH (1998)
- RegEM

$$T_t = \int_{NH} \sum_k \lambda_k u_{t,k} v_k \, dA + \varepsilon_t$$

PACIFIC CLIMATE IMPACTS CONSORTIUM

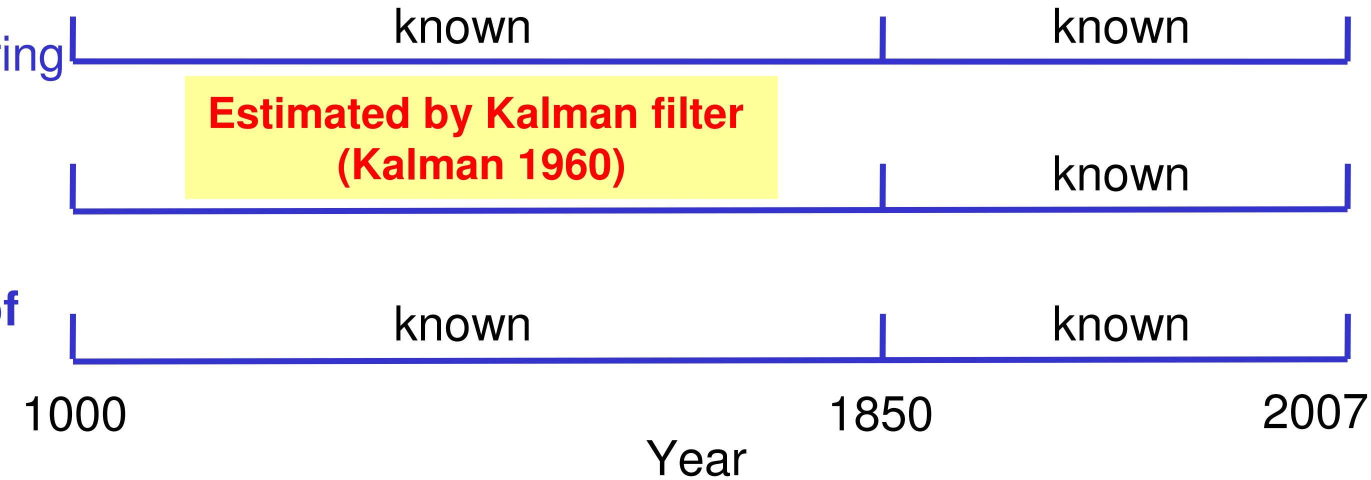
University of Victoria

State-space model

Proxy data (P_t)
e.g., thickness of tree ring

NH temperature (T_t)

Estimated influence of external forcings (F_t)



UNKNOWN !

Observation equation: $$\mathbf{P}_t = \mathcal{A}\mathbf{T}_t + e_t, \qquad e_t \sim \mathrm{N}(0, \mathcal{R})$$

State equation: $$\mathbf{T}_t = \phi\mathbf{T}_{t-1} + \delta\mathbf{F}_t + w_t, \qquad w_t \sim \mathrm{N}(0, \mathcal{Q})$$

PACIFIC CLIMATE IMPACTS CONSORTIUM

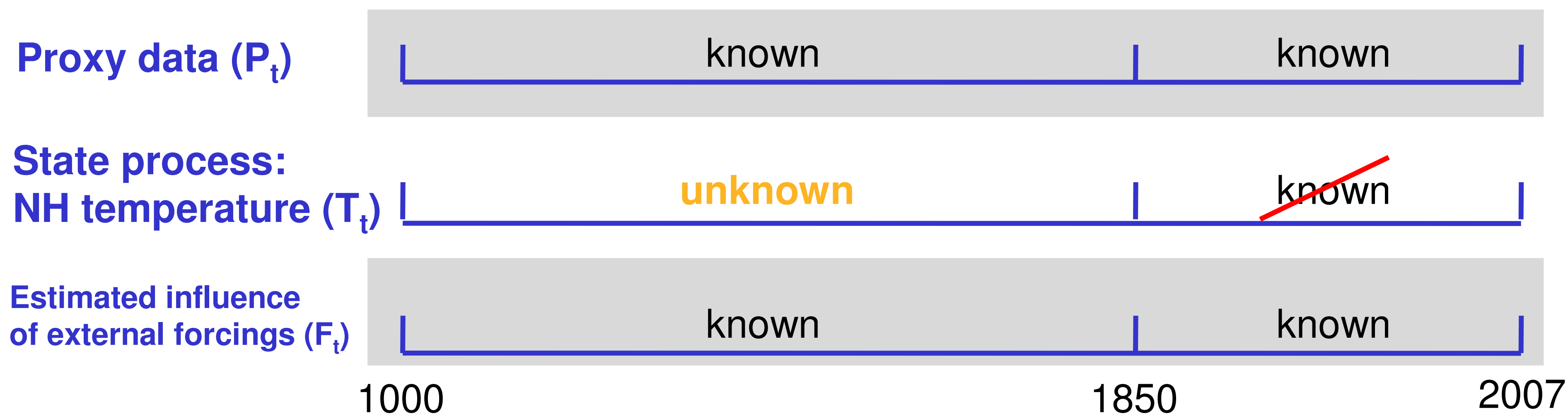
University of Victoria

Standard parameter estimation approach (STD)

- Maximum likelihood of state process given proxies and estimated responses to forcing.

$$\ell_{\mathbf{P}}(\Theta) = -0.5\sum_{t=1}^{N}\ln\left|\mathcal{A}\mathbf{S}_{t|t-1}\mathcal{A}^{\mathrm{T}} + \mathcal{R}\right| - 0.5\sum_{t=1}^{N}(\mathbf{P}_t - \mathcal{A}\mathbf{T}_{t|t-1})^{\mathrm{T}}(\mathcal{A}\mathbf{S}_{t|t-1}\mathcal{A}^{\mathrm{T}} + \mathcal{R})^{-1}(\mathbf{P}_t - \mathcal{A}\mathbf{T}_{t|t-1})$$

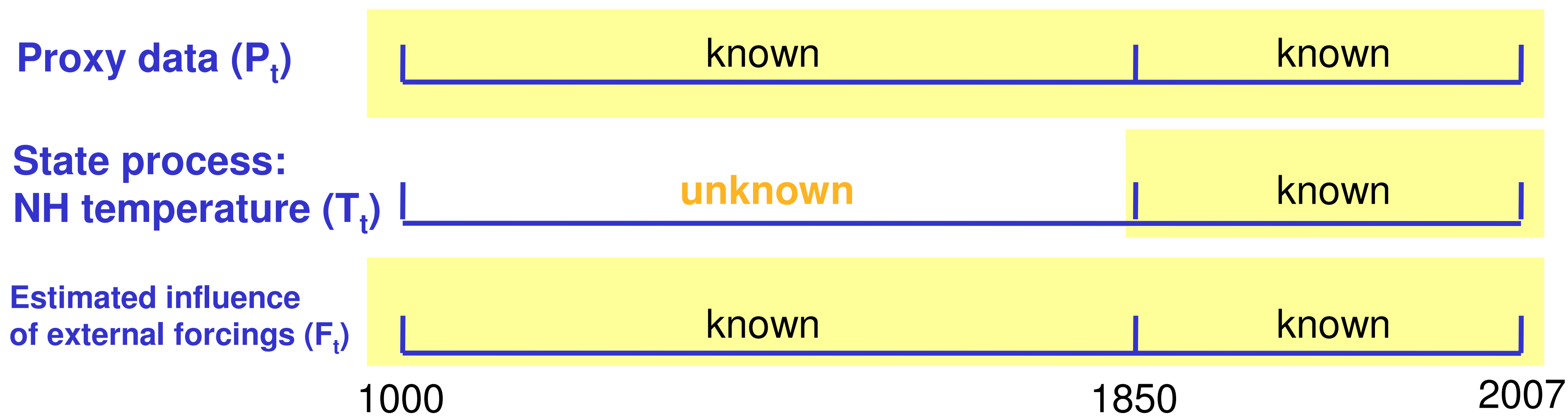
Kalman filter estimates: non-linear function of parameters



Improved parameter estimation technique (ALL)

$$
\begin{aligned}
-2\,\ell_{\mathbf{P},\mathbf{T}}(\Theta) = \sum_{t=1}^{n}\Big[&\ln|\mathcal{A}\mathbf{S}_{t|t-1}\mathcal{A}^{\mathrm{T}}+\mathcal{R}| + (\mathbf{P}_t-\mathcal{A}\mathbf{T}_{t|t-1})^{\mathrm{T}}(\mathcal{A}\mathbf{S}_{t|t-1}\mathcal{A}^{\mathrm{T}}+\mathcal{R})^{-1}(\mathbf{P}_t-\mathcal{A}\mathbf{T}_{t|t-1})\Big]\\
&+ m\ln|\mathcal{R}| + \sum_{t=n+1}^{N}(\mathbf{P}_t-\mathcal{A}\mathbf{T}_t)\mathcal{R}^{-1}(\mathbf{P}_t-\mathcal{A}\mathbf{T}_t)^{\mathrm{T}}\\
&+ (m-1)\ln|\mathcal{Q}| + \sum_{t=n+2}^{N}(\mathbf{T}_t-\phi\mathbf{T}_{t-1}-\delta\mathbf{F}_t)\mathcal{Q}^{-1}(\mathbf{T}_t-\phi\mathbf{T}_{t-1}-\delta\mathbf{F}_t)^{\mathrm{T}}\\
&+ \ln|\phi\mathbf{S}_{n|n}\phi^{\mathrm{T}}+\mathcal{Q}|\\
&+ (\mathbf{T}_{n+1}-\phi\mathbf{T}_{n|n}-\delta\mathbf{F}_{n+1})(\phi\mathbf{S}_{n|n}\phi^{\mathrm{T}}+\mathcal{Q})^{-1}(\mathbf{T}_{n+1}-\phi\mathbf{T}_{n|n}-\delta\mathbf{F}_{n+1})^{\mathrm{T}}.
\end{aligned}
$$

Kalman filter estimates



PACIFIC CLIMATE IMPACTS CONSORTIUM

University of Victoria

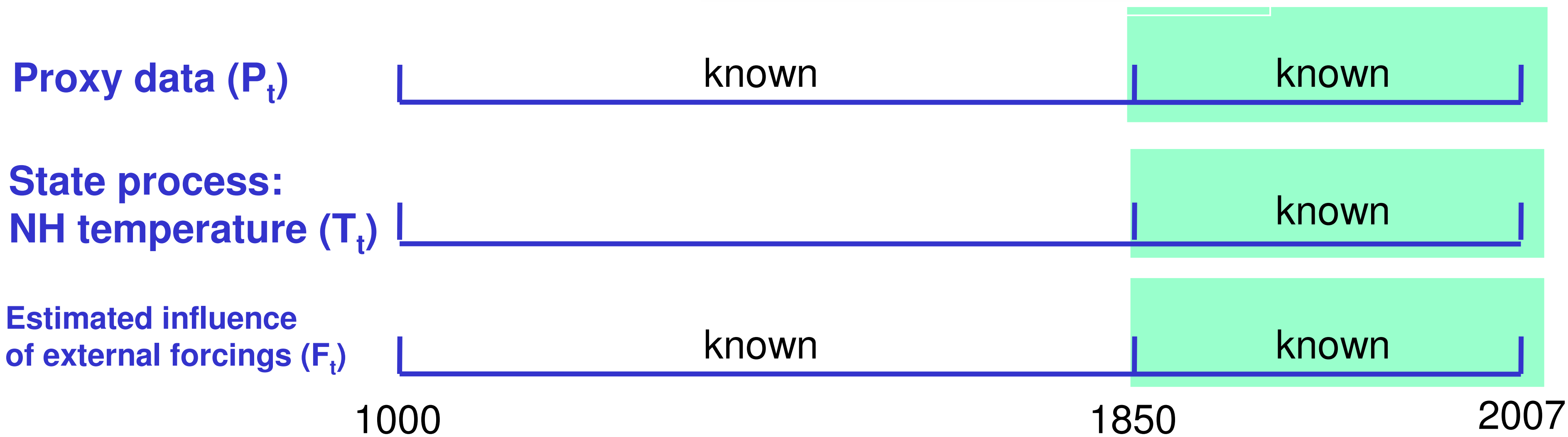
Competing simple parameter estimation technique (CAL)

- Uses only calibration period data.

$$-2\,\ell_{\mathbf{c}}(\Theta) = \sum_{t=n+1}^{N}\left[\ln\mathcal{R} + (\mathbf{P}_t - \mathcal{A}\mathbf{T}_t)^2/\mathcal{R}\right] + \sum_{t=n+2}^{N}\left[\ln\mathcal{Q} + (\mathbf{T}_t - \phi\mathbf{T}_{t-1} - \delta\mathbf{F}_t)^2/\mathcal{Q}\right]$$

- MLEs can be solved explicitly, for example,

$$\hat{\mathcal{A}} = \left(\sum_{t=n+1}^{N}\mathbf{T}_t^2\right)^{-1}\sum_{t=n+1}^{N}\mathbf{T}_t\mathbf{P}_t \qquad \hat{\mathcal{R}} = m^{-1}\sum_{t=n+1}^{N}(\mathbf{P}_t - \hat{\mathcal{A}}\mathbf{T}_t)^2$$



Comparing estimator properties

- Likelihood too complicated to derive asymptotic properties in all cases ➔ used Monte-Carlo
- Generate data under assumed model:

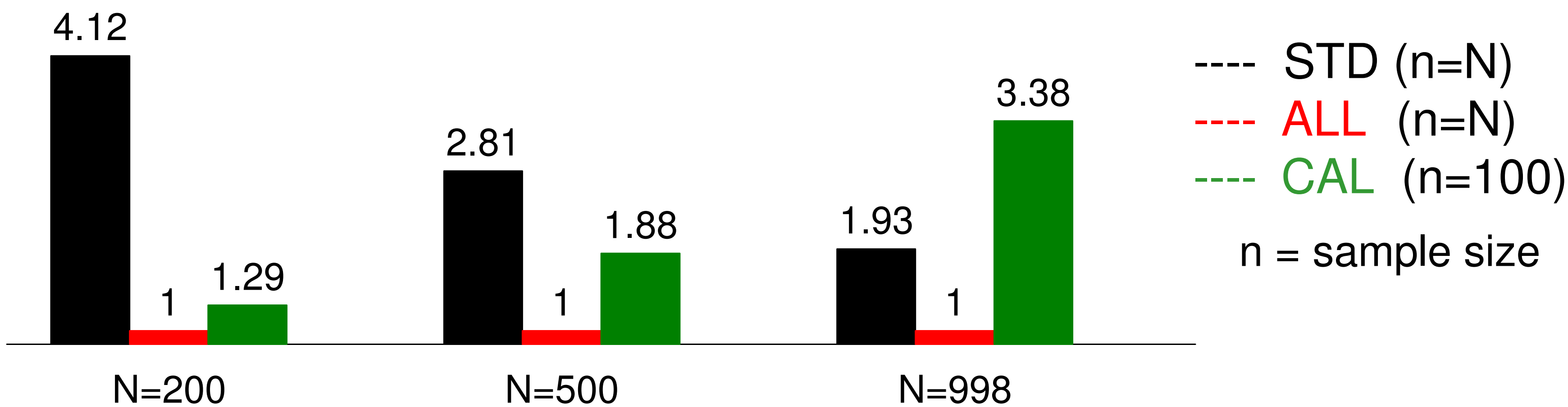
$$\mathrm{P}_t = A\,\mathrm{T}_t + e_t, \qquad e_t \sim N(0,R)$$
$$\mathrm{T}_t = \phi\,\mathrm{T}_{t\text{-}1} + \delta\,F_t + w_t, \quad w_t \sim N(0,Q)$$

  - Specified reasonable values for A, ϕ, δ, R and Q.
  - Used F_t from a simple EBM of the climate.
  - Generated noise terms e_t and w_t from normal.

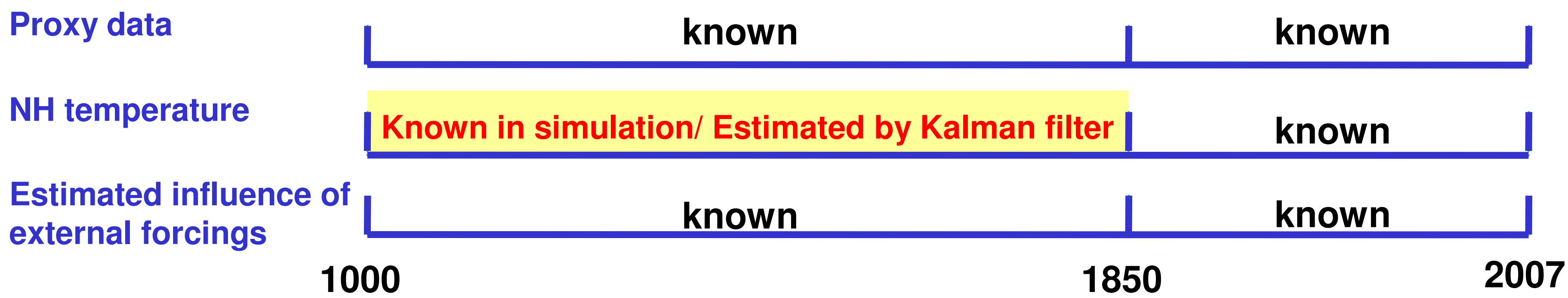
Relative efficiency of persistence parameter

Smaller value in efficiency= smaller estimator variance
= better estimator

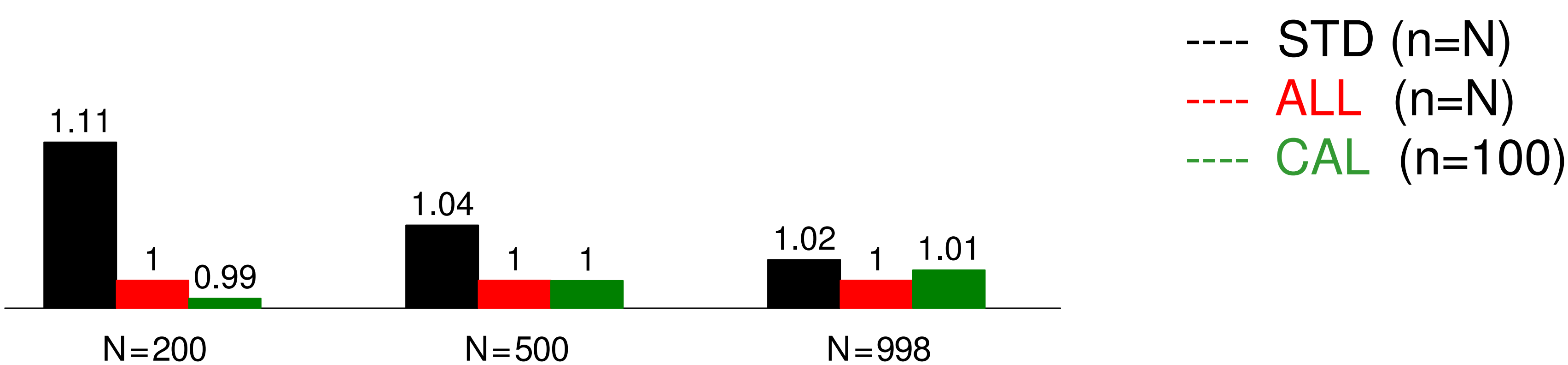
Estimated efficiency for ϕ (relative to ALL)



Reconstruction error



Reconstruction error (relative to ALL)



Smaller error = better approach

PACIFIC CLIMATE IMPACTS CONSORTIUM

University of Victoria

Robustness

$$e_t = \alpha e_{t-1} + \kappa_t, \ \kappa_t \sim N(0, R)$$

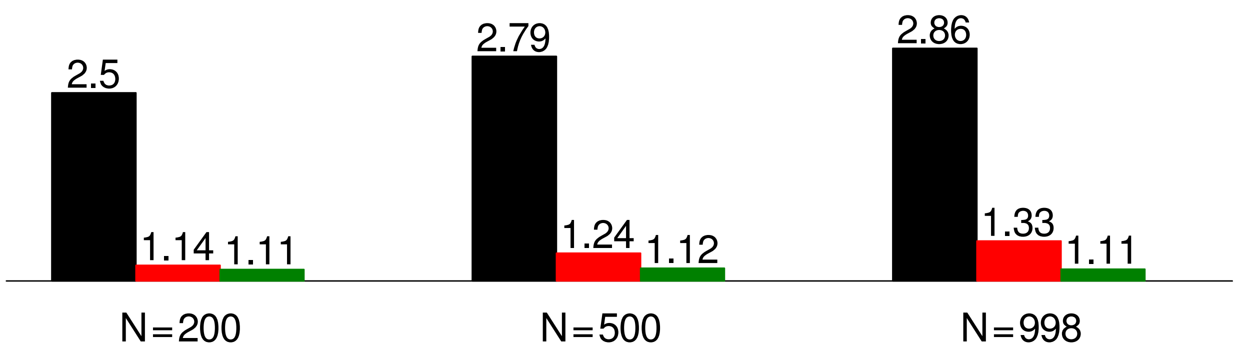
$$P_t = A\,T_t + e_t,$$ ~~$e_t \sim N(0,R)$~~

$$T_t = \phi\, T_{t-1} + \delta F_t + w_t,$$

Estimators from CAL are asymptotically unbiased!

Change in reconstruction error
(relative to iid case)

Smaller change = more robust



---- STD (n=N)
---- ALL (n=N)
---- CAL (n=100)

Summary

- STD approach should be avoided.
  - Larger bias.
  - Lower efficiency.
  - Less robust.
- CAL approach is more robust
  - Reconstruction errors similar to ALL approach
  - Resistant to misspecification of state equation
  - MLEs easily obtained
- ALL approach if desire is to estimate state equation parameters

Evaluating the reconstruction techniques

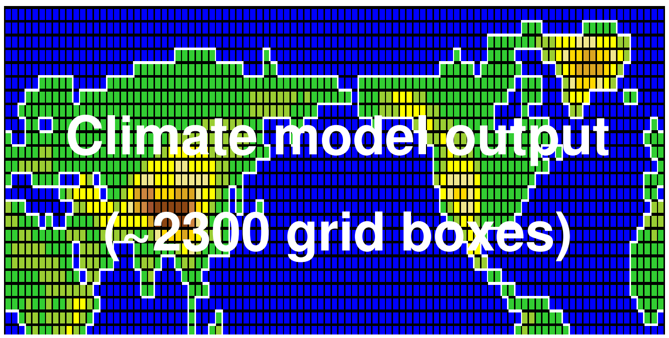
Photo: F. Zwiers

Evaluating Reconstruction techniques

- von Storch et al (2004) proposed using long climate simulations
- Idea was to
  - Sample model output at locations coinciding with proxies
  - Degrade by adding white noise
  - Apply reconstruction method
  - Compare with the known model simulated hemispheric mean temperature
- There are nuances to consider such as,
  - how much noise should be added?
  - what colour of noise should be added?
  - should one detrend prior to diagnosing the obs/proxy relationship?
  - what part of the spectrum should be used?
- The latter three questions are all broadly equivalent

Proposed evaluation technique

- Proposed by von Storch et al. (2004)



Simulated NH mean temperature time series

1) Add noise to 100 grid-box temperature series

3) *Compare*

100 pseudo-proxy series

Reconstructed NH mean temperature time series

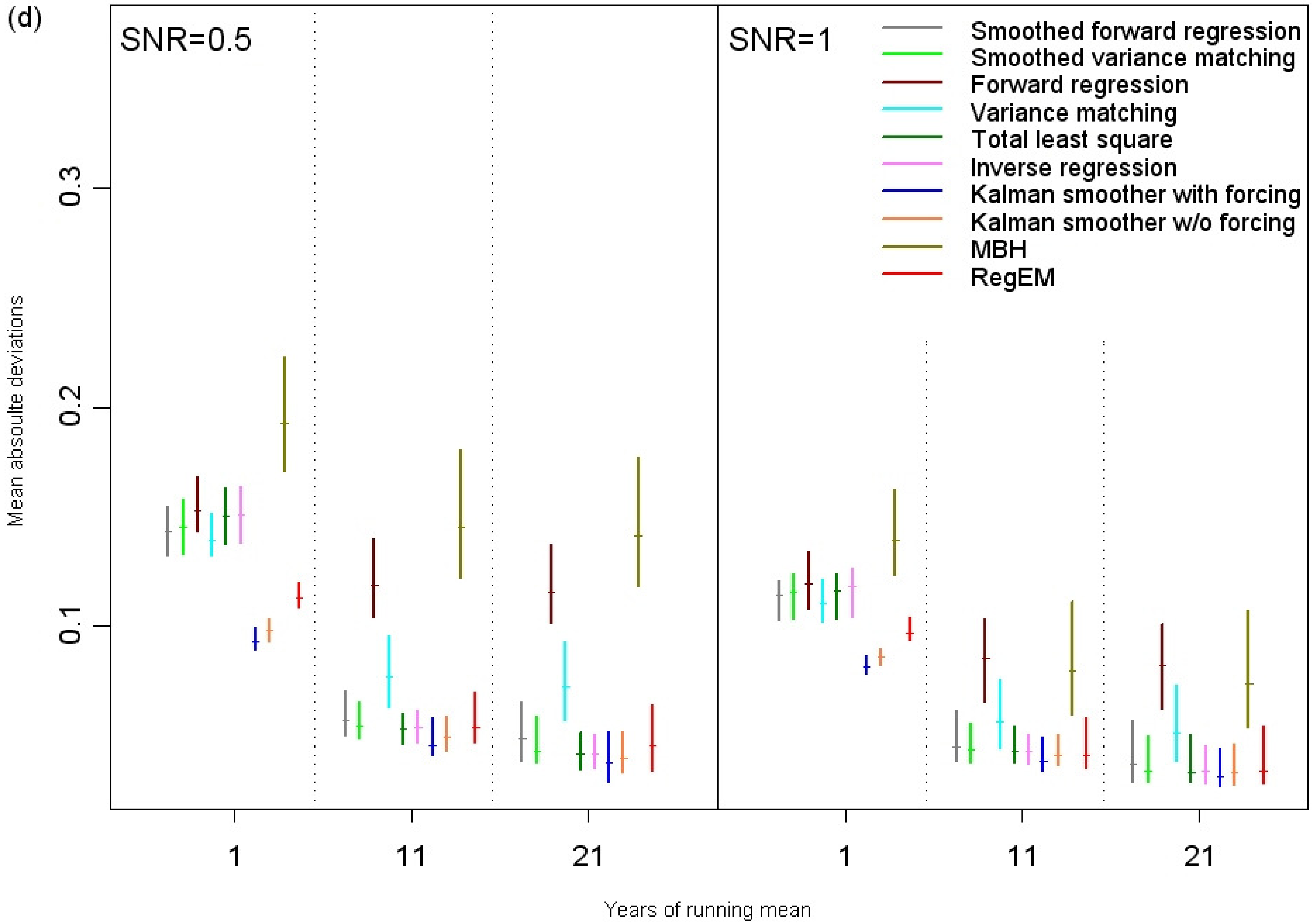
2) *Apply reconstruction method*

Some details

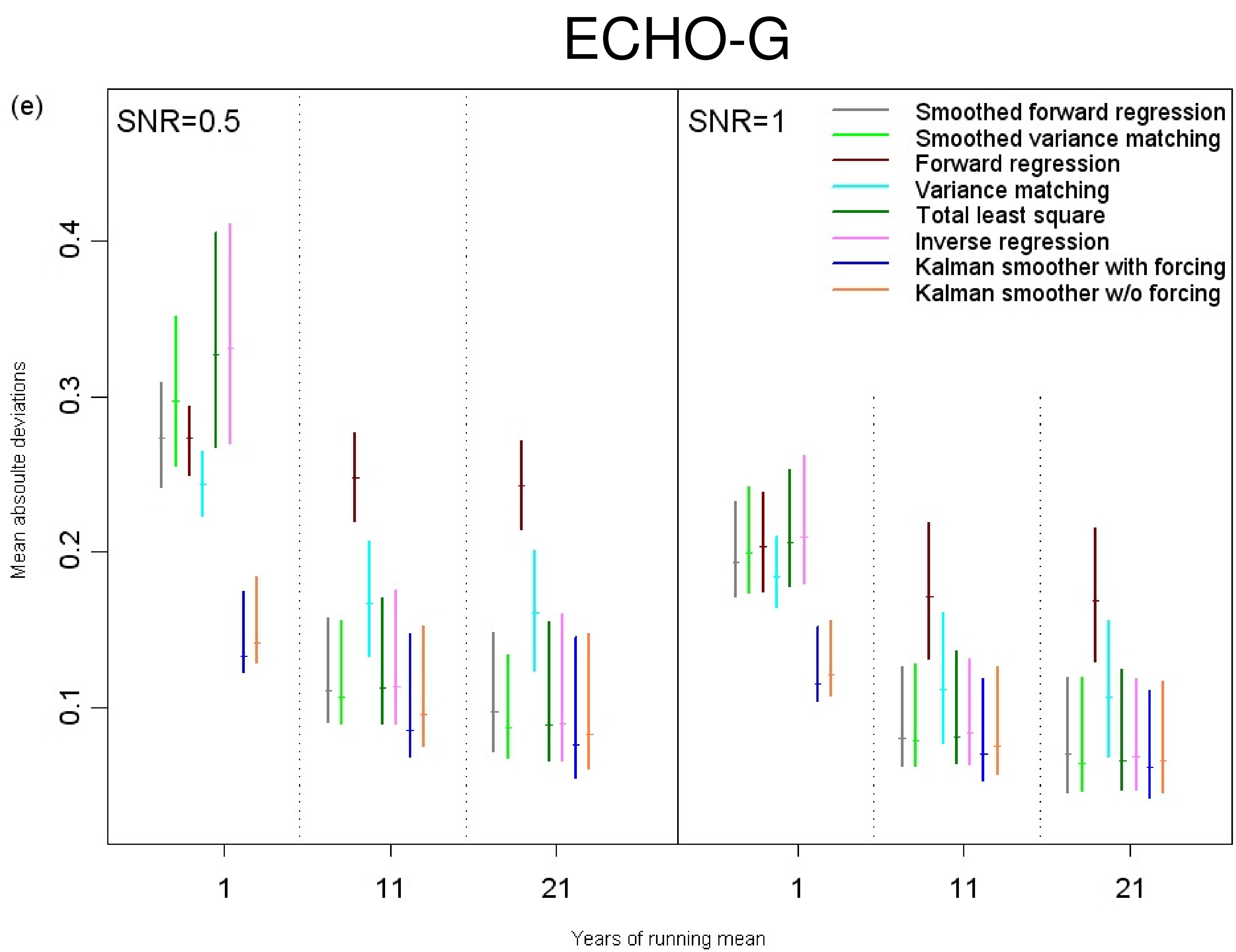
- We did this
  - Two different climate models
  - Two signal to noise ratios
  - White and red noise
  - Different sizes of proxy networks
  - Different lengths of calibration period
  - Effects of detrending prior to calibration
- We did NOT condition on a fixed proxy network
  - Repeated reconstruction 1000 times
    - A different proxy network at each time
    - A different realization of added noise each time

100 pseudo proxies – 1860-1970 calibration – mean abs deviation

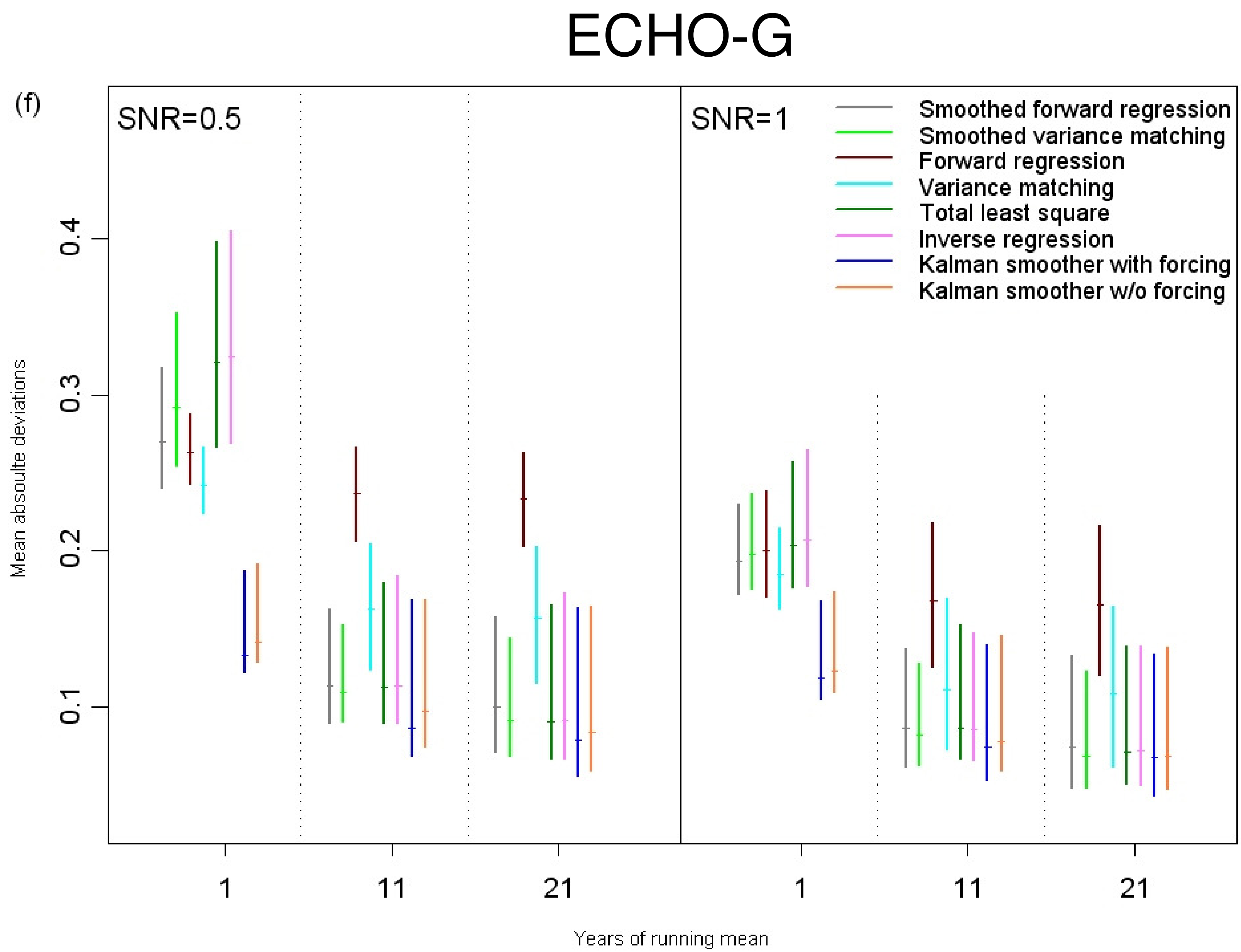
ECHO-G



15 pseudo proxies – 1860-1970 calibration – mean abs deviation

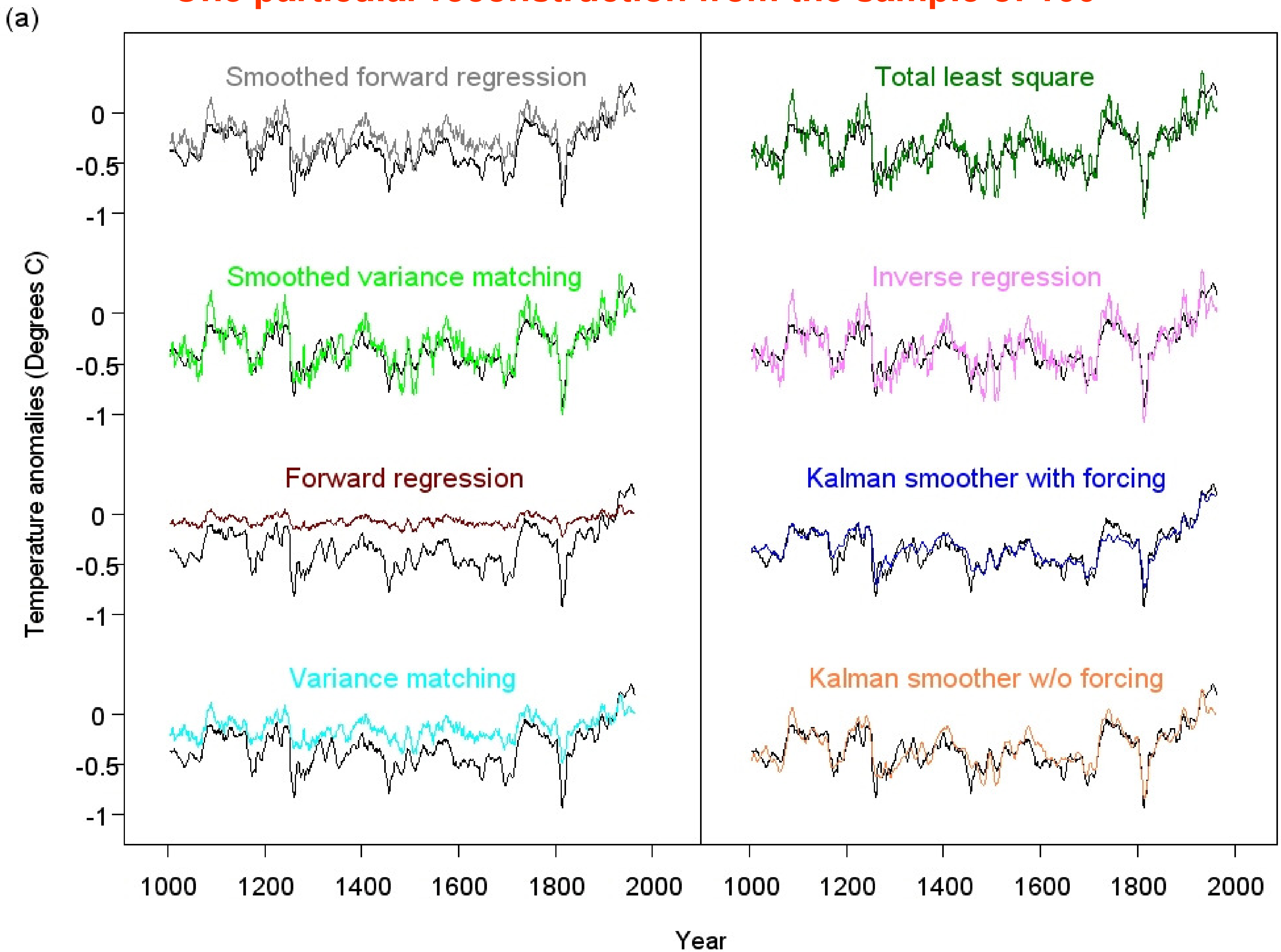


15 pseudo proxies – 1880-1960 calibration – mean abs deviation



15 point network – 11 year moving average – CSM - SNR = 0.5

One particular reconstruction from the sample of 100



PACIFIC CLIMATE IMPACTS CONSORTIUM

University of Victoria

Conclusions

- Some reconstruction methods to should be avoided
- Several other methods compare well, particularly at low frequency
- Assessment is not sensitive to which “reality” one compares against
- CFR methods do not have a particular predilection to producing hockey stick like reconstructions
- The new Mann et al CFR technique works well, but CFR does not appear to be consistently better that CPS
- Kalman filter approach appears to produce consistently good results across time scales
- There is probably greater sensitivity to the choice of proxy than to the choice of reconstruction method (see also Juckes et al, 2007, *Climate of the Past Discussions*, **vol 2**)

Two Extensions using Kalman smoother …

State space model

$$P_t = \beta T_t + \varepsilon_t$$

$$T_t = \varphi T_{t-1} + \delta F_t + \omega_t$$

where

$$F_t = \alpha_{Sol} r(Sol_t) + \alpha_{Vol} r(Vol_t) + \alpha_G r(GHG_t) + \alpha_S r(Sul_t)$$

- Data assimilation (of sorts) using a climate model to assimilate proxy information and observed hemispheric temperatures
- Fitting the state space model provides estimates of coefficients and their uncertainty ➔ allows "***detection***"
- Integrate climate model forward in time ➔ allows "***prediction***"

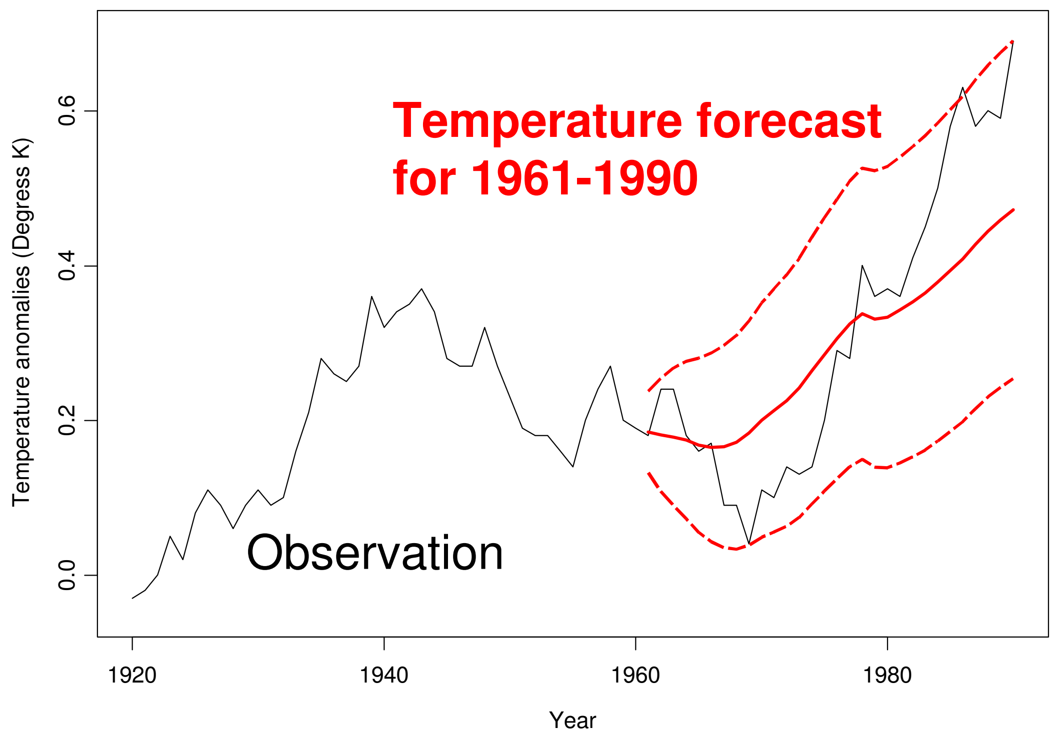
Results …

Climate change detection:

- Using real-world proxy data (1510-1960):
  - Effect of greenhouse gas, aerosol and volcanic forcing has significantly affected our past climate.
  - Consistent with other detection studies.

Temperature prediction:

- Result with real-world proxy data (1510-1960):




The End
Photo: F.Zwiers

References

- Lee, T.C.K., F.W. Zwiers and M. Tsao, 2008: Evaluation of millennial proxy reconstruction methods. *Climate Dynamics*, **31**, 263-281, doi 10.1007/s00382-007-0351-9.
- Lee, T.C.K., M. Tsao, F.W. Zwiers, 2010: State-space model for proxy-based millennial reconstruction. *Canadian Journal of Statistics*, **38**, 488-505.