DA-based Attribution:
A tentative illustration on stratospheric cooling

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Outline of the presentation

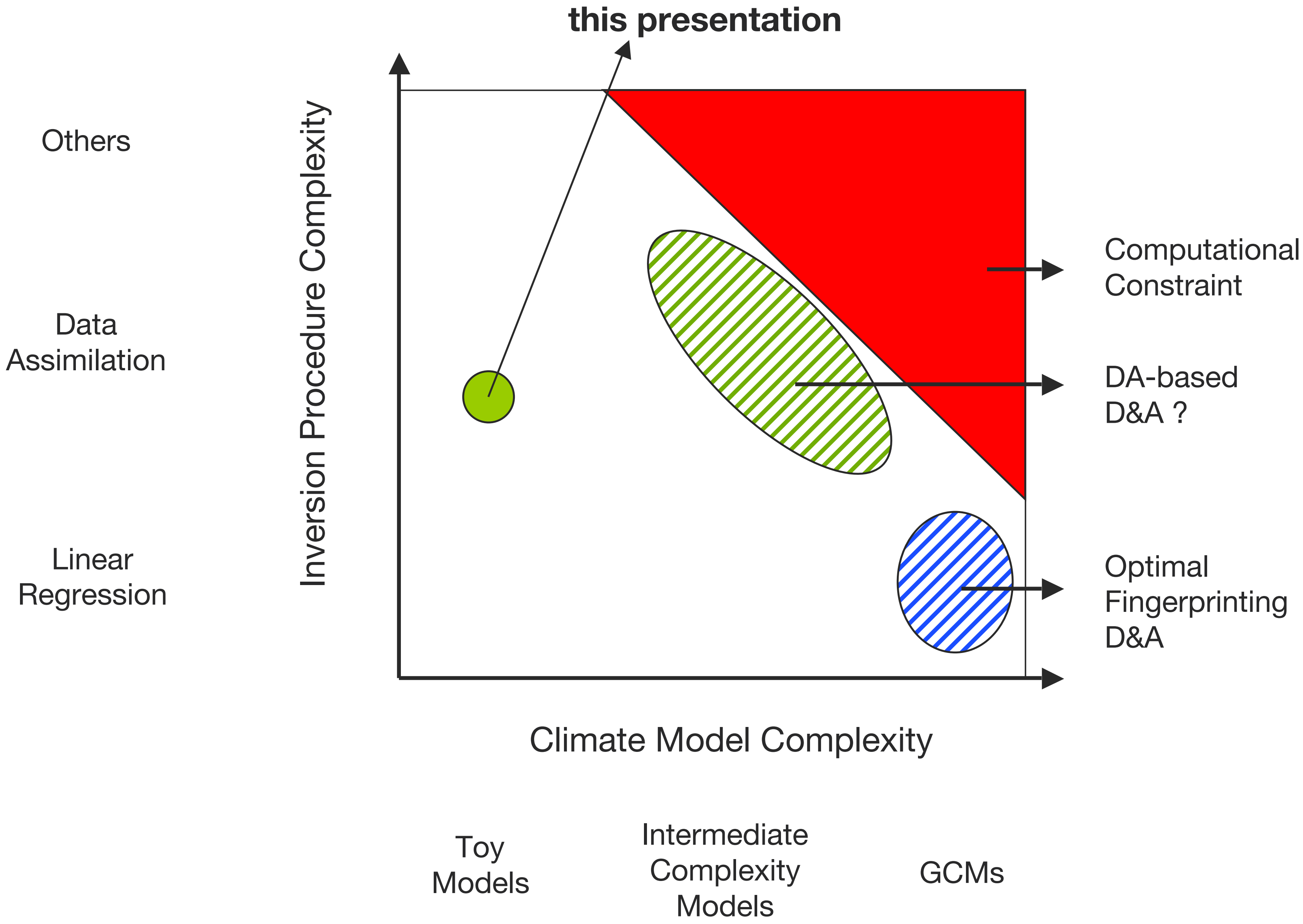
- Motivation and purpose
- Model description
- Inversion
- Confounding factors
- Optimal fingerprinting
- Metamodeling
- Conclusions

Motivation and approach followed

- Provide a tentative illustration of Data Assimilation-based Attribution.
  - ‘Methodological lab’: initial testing of the DADA idea
  - ‘Realistic’ historical D&A case: stratospheric cooling.
  - Purely synthetic data (simulated observations only).

- Proposed steps:
  - Implementation of a 1D toy model ($n = 10^1$)
  - Parameters describe forcings.
  - Evaluation of parameters based on observations
  - Using the Augmented-State EKF.

Data Assimilation for Detection and Attribution



What is parameter estimation?

- ✓ Numerical models have a dynamic core and parameterizations of the ”physics”.
- ✓ In both components there are parameters that have some impact upon the model performance.
- ✓ Most of this parameters arise from the formulation of numerical schemes and the simplifying assumptions made in parameterizations. So these parameters are intrinsically unknown and have to be tuned in order to get a good performance.
- ✓ Some external forcings may also be treated as parameters in the model equations.

$$\frac{dx}{dt} = \underbrace{f(x,t,p_1)}_{\text{Dynamic core}} + \underbrace{par(x,t,p_2)}_{\text{Parameterizations (model physics)}} + \underbrace{f}_{\text{Forcing}}$$

Sequential parameter estimation

- "**State augmentation**" method – uncertain parameters are treated as additional state variables.
- Example: one unknown parameter μ

$$\bar{x}_k = \begin{pmatrix} x_k \\ \mu_k \end{pmatrix} = \begin{pmatrix} F(x_{k-1}, \mu_{k-1}) \\ \mu_{k-1} \end{pmatrix} + \begin{pmatrix} \epsilon_k \\ \epsilon^{\mu}_{k-1} \end{pmatrix}$$

$$y^o_k = \begin{pmatrix} H & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x_k \\ \mu_k \end{pmatrix} + \epsilon^0 = \bar{H}\bar{x}_k + \epsilon^0$$

$$\bar{x}^a_k = \bar{x}^f_k + \bar{K}(y^o_k - \bar{H}\bar{x}^f_k); \quad \bar{K} = \bar{P}^f \bar{H}^T (\bar{H}\bar{P}^f \bar{H}^T + R)^{-1}$$

- **The parameters are not directly observable, but** the **cross-covariances** drive parameter changes from innovations of the state:

$$\bar{P}^f = \begin{pmatrix} P^f_{xx} & P^f_{x\mu} \\ P^f_{\mu x} & P^f_{\mu\mu} \end{pmatrix}; \quad \bar{K} = \begin{pmatrix} P^f_{xx} H^T \\ P^f_{\mu x} H^T \end{pmatrix} (H P^f_{xx} H^T + R)^{-1}$$

- Parameter estimation is always a **nonlinear problem**, even if the model is **linear** in terms of the model state: use **Extended Kalman Filter (EKF).**

Anterior works

- DA-based parameter estimation (Juan and Manuel’s presentations)
  - large and increasing number of studies and applications across Geophysics.

- DA-based reconstruction (Marc’s presentation):
  - of a source (pollutant)
  - from the observation of its effects
  - using a physical model (advection)

- DA-based reconstruction of radiative forcings:
  - Annan 2005: an illustration using the Lorenz model

Anterior works: Annan 2005

- Illustrative approach based on a toy's model.
- Forward model: the forced Lorenz model.

$$\begin{cases} x' = \sigma(y - x) + f\cos\theta \\ y' = rx - y - xz + f\sin\theta \\ z' = xy - bz \end{cases}$$

- Unobserved: forcing magnitude coefficients.

$$f(t) = \alpha_1 f_1(t) + \alpha_2 f_2(t) + \alpha_3 f_3(t) + \alpha_4 f_4(t)$$

- Inversion procedure: Augmented-State EnKF.
- Result: it works (convergence towards true value)

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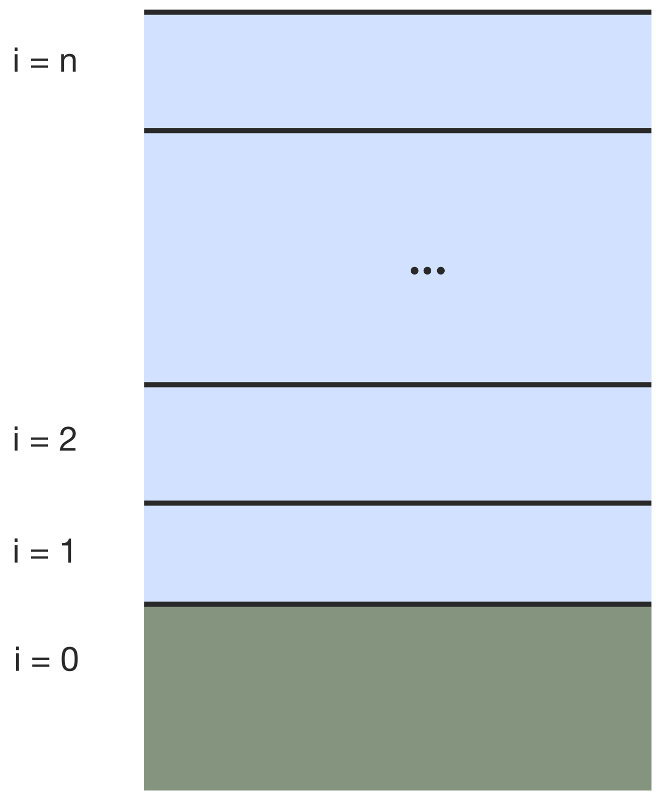
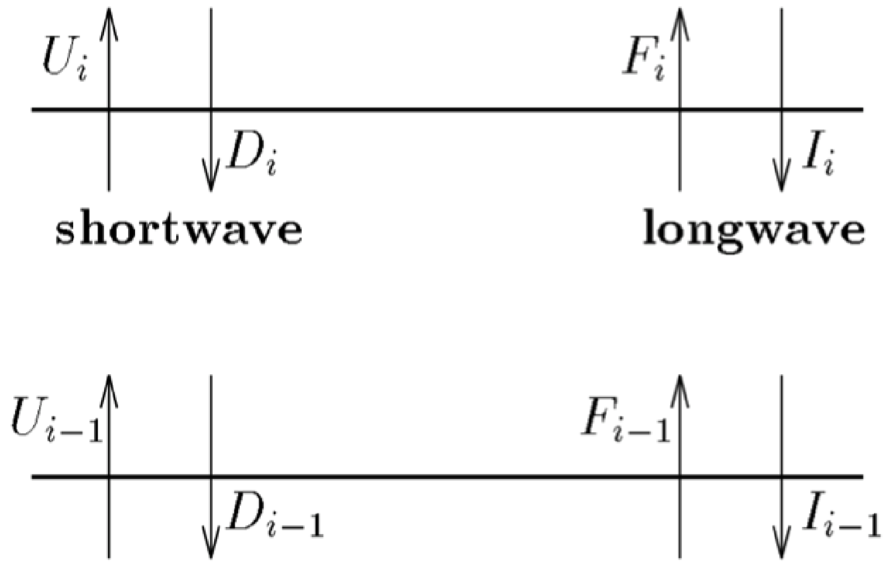
Description of the model

- A 1D atmospheric column radiative model (Li et al. [1997])
- Schematic diagrams:

n atmospheric layers



solar and terrestrial radiative fluxes in layer i



Description of the model

- Model equations for layers i = 1, 2, … , n-1
- Physical properties of fluxes between two adjacent layers:

$$\Rightarrow \begin{cases} I_i = I_{i+1}t_{i+1} + (1 - t_{i+1})\sigma T_{i+1}^4 \\ F_i = F_{i-1}t_i + (1 - t_i)\sigma T_i^4 \\ U_i = U_{i-1}\tau_i + D_i\rho_i \\ D_i = D_{i+1}\tau_{i+1} + U_i\rho_{i+1} \end{cases}$$

t: longwave transmissivity

τ: shortwave transmissivity

ρ: shortwave reflectivity

- Radiative equilibrium within layer i:

$$\Rightarrow \quad (D_i - D_{i-1}) - (I_i - I_{i-1}) + (U_i - U_{i-1}) - (F_i - F_{i-1}) = 0$$

Description of the model

- Model equations for top and bottom layers (i = 0 and n):

- Physical relationships at boundaries:

$$\Rightarrow \begin{cases} D_n = S/4 \\ I_n = 0 \\ F_0 = \sigma T_0^4 \\ U_0 = D_0 \rho_0 \end{cases}$$

S: solar constant

ρ_0: surface albedo

- Dynamics entirely driven by surface heat take-up:

$$\Rightarrow \quad D_0 - I_0 + U_0 - F_0 = c\frac{dT_0}{dt}$$

Resolution of the model

- Static resolution

- Prescribed: time-constant spatial patterns of ρ, τ and t

$$\Downarrow$$

- Result: equilibrium spatial pattern **T*** of temperature (exact expression)

$$\mathbf{T}^* = G(\rho, \tau, t)$$

Closed form resolution

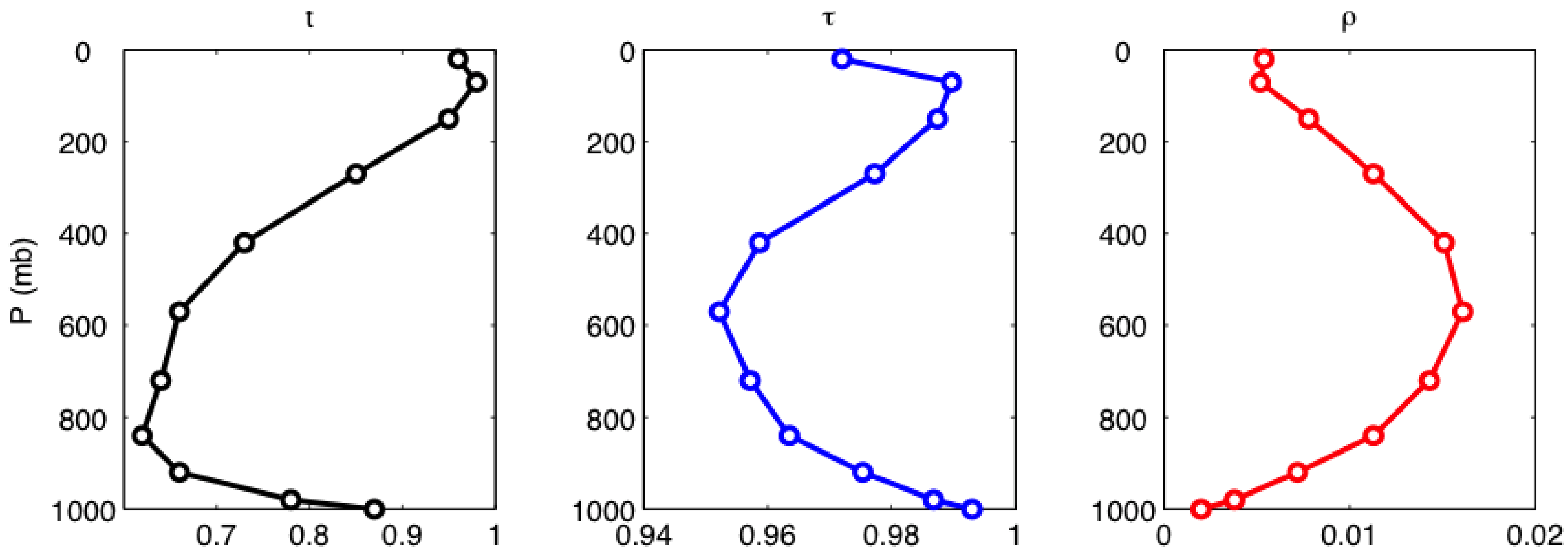
$$G(\rho, t, \tau) = (\mathbf{K}^{-1} E/\sigma)^{1/4}$$

$$\mathbf{K} = \begin{cases} k_{00} = -1, \\ k_{ii} = -2(1-t_i), \quad i = 1, 2, ..., n, \\ k_{01} = (1-t_1), \\ k_{0j} = (1-t_j) \prod_{l=1}^{l=j-1} t_l, \quad j = 2, 3, ..., n, \\ k_{i,i+1} = (1-t_i)(1-t_{i+1}), \quad i = 1, 2, ..., n, \\ k_{ij} = (1-t_i)(1-t_j) \prod_{l=i+1}^{l=j-1} t_l, \quad i = 1, 2, ..., n \ (j > i+1) \end{cases}$$

$$E = \begin{cases} E_0 = S/4(1-\rho_0) \prod_{l=1}^{l=n} \tau_l, \\ E_i = S/4(\prod_{l=i+1}^{l=n} \tau_l)\{1 - \tau_i - \rho_i + \tau_i(1-\tau_i)[\sum_{l=0}^{l=i-1} (\rho_l \prod_{m=l+1}^{m=i-1} \tau_m^2)]\} \end{cases}$$

Resolution of the model

- Static resolution
- Prescribed: time-constant spatial patterns of ρ, τ and t (from IPSL model - Li et al. [1997])

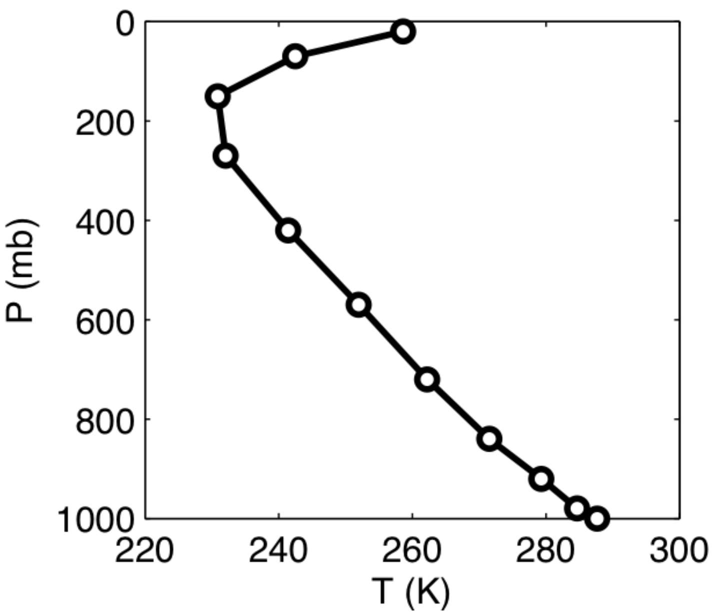


Resolution of the model

- Static resolution

- Result: equilibrium spatial pattern **T*** of temperature

$$\mathbf{T}^* = G(\rho, \tau, t)$$



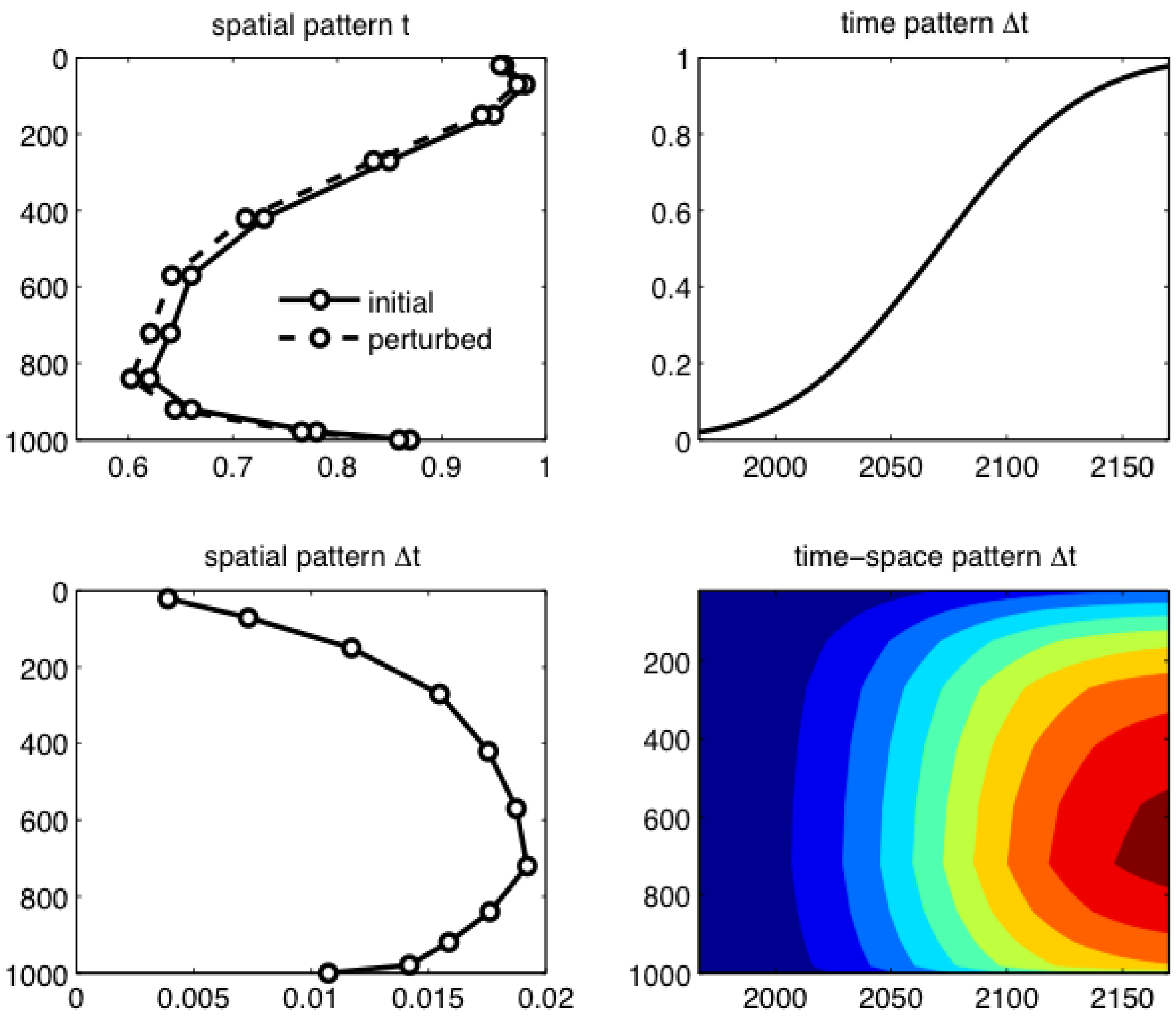
Resolution of the model

- Dynamic resolution
- Prescribed: time-trajectories of the spatial patterns of ρ, τ and t
- Result: time-trajectory of the spatial pattern of temperature (discretization)

$$\mathbf{T}_{t+1} = F(\mathbf{T}_t; \rho_t, \tau_t, t_t)$$

Resolution of the model

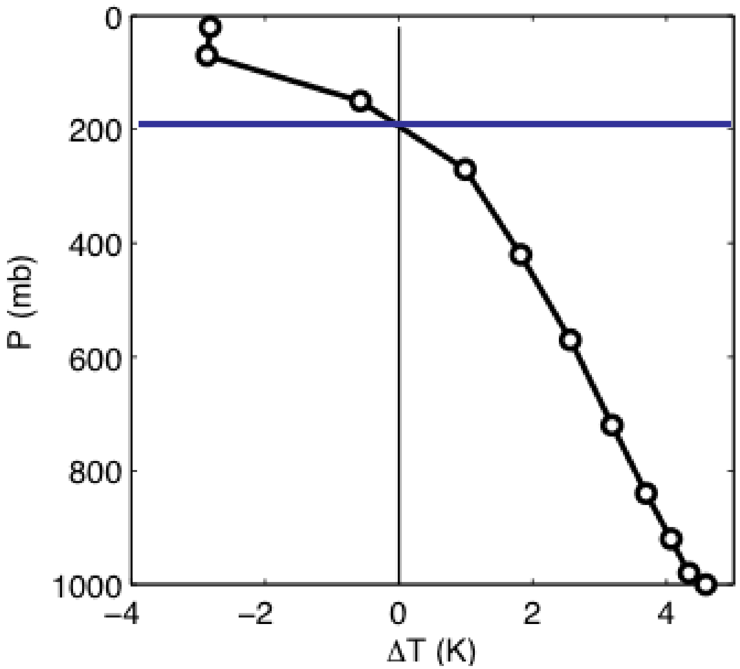
- Prescribed: time-trajectory of the spatial pattern of t (GHG forcing)



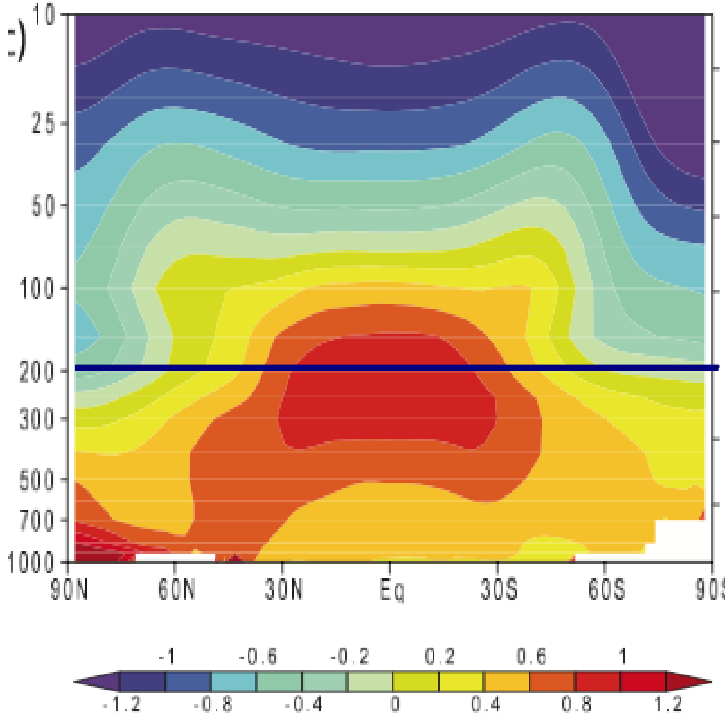
Resolution of the model

- Result: time-trajectory of the spatial pattern of temperature (GHG forcing only)

1D model

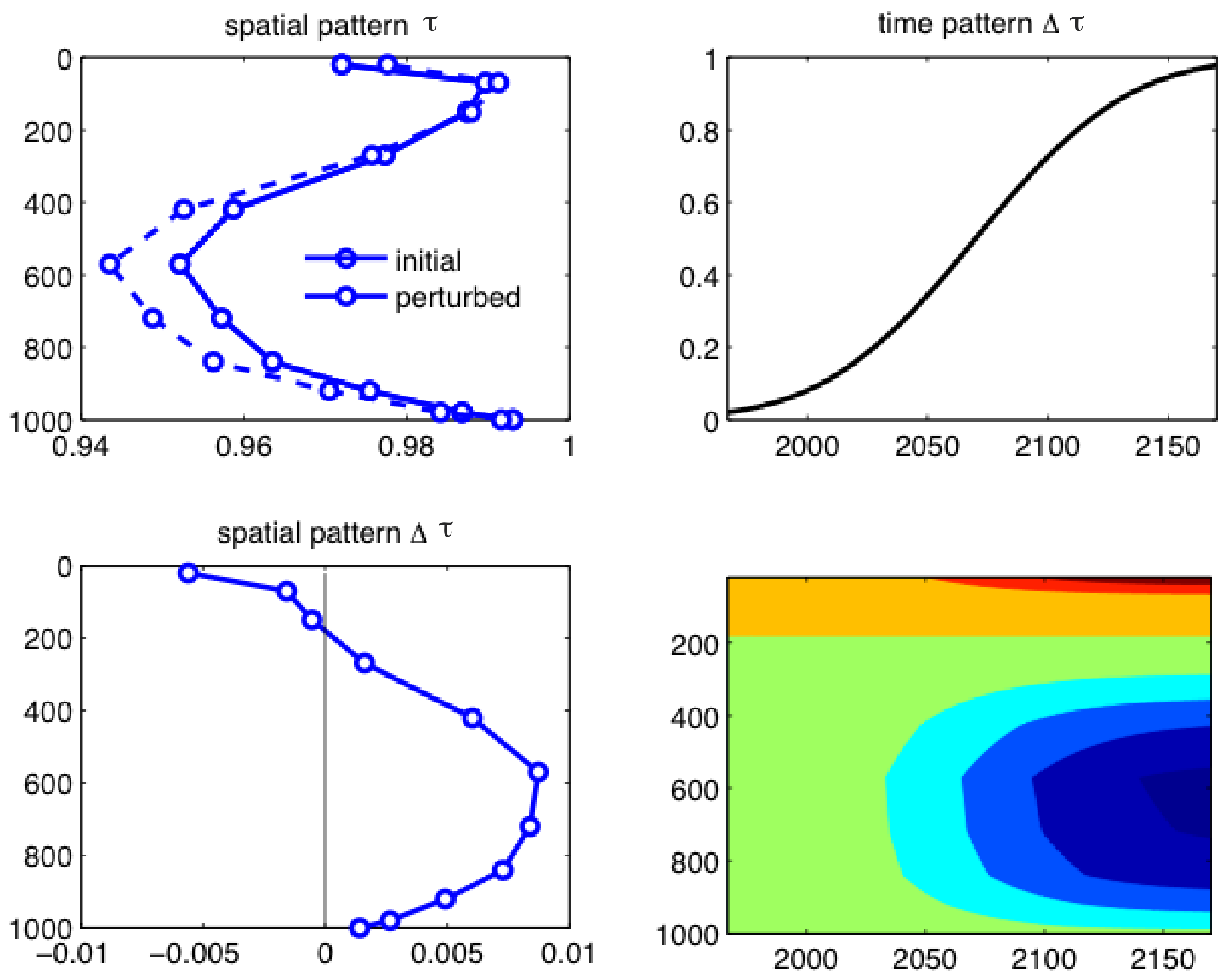


PCM model (AR4 [2007])



Resolution of the model

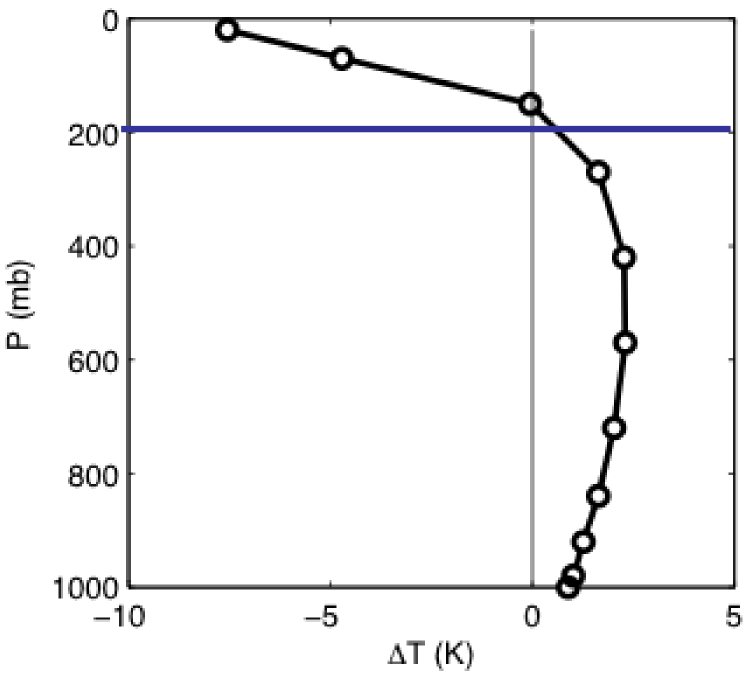
- Prescribed: time-trajectory of the spatial pattern of τ (O_3 forcing)



Resolution of the model

- Result: time-trajectory of the spatial pattern of temperature (GHG forcing only)

1D model



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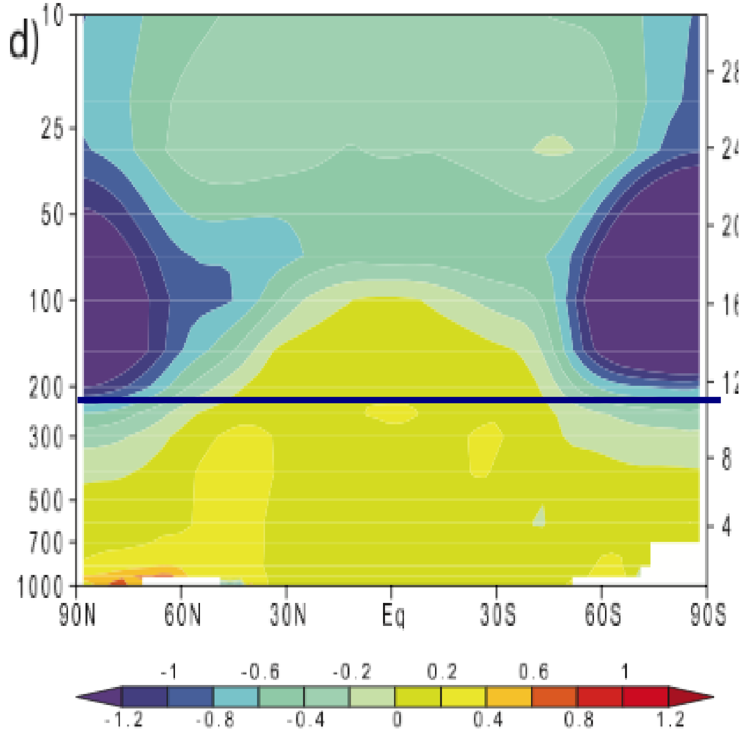
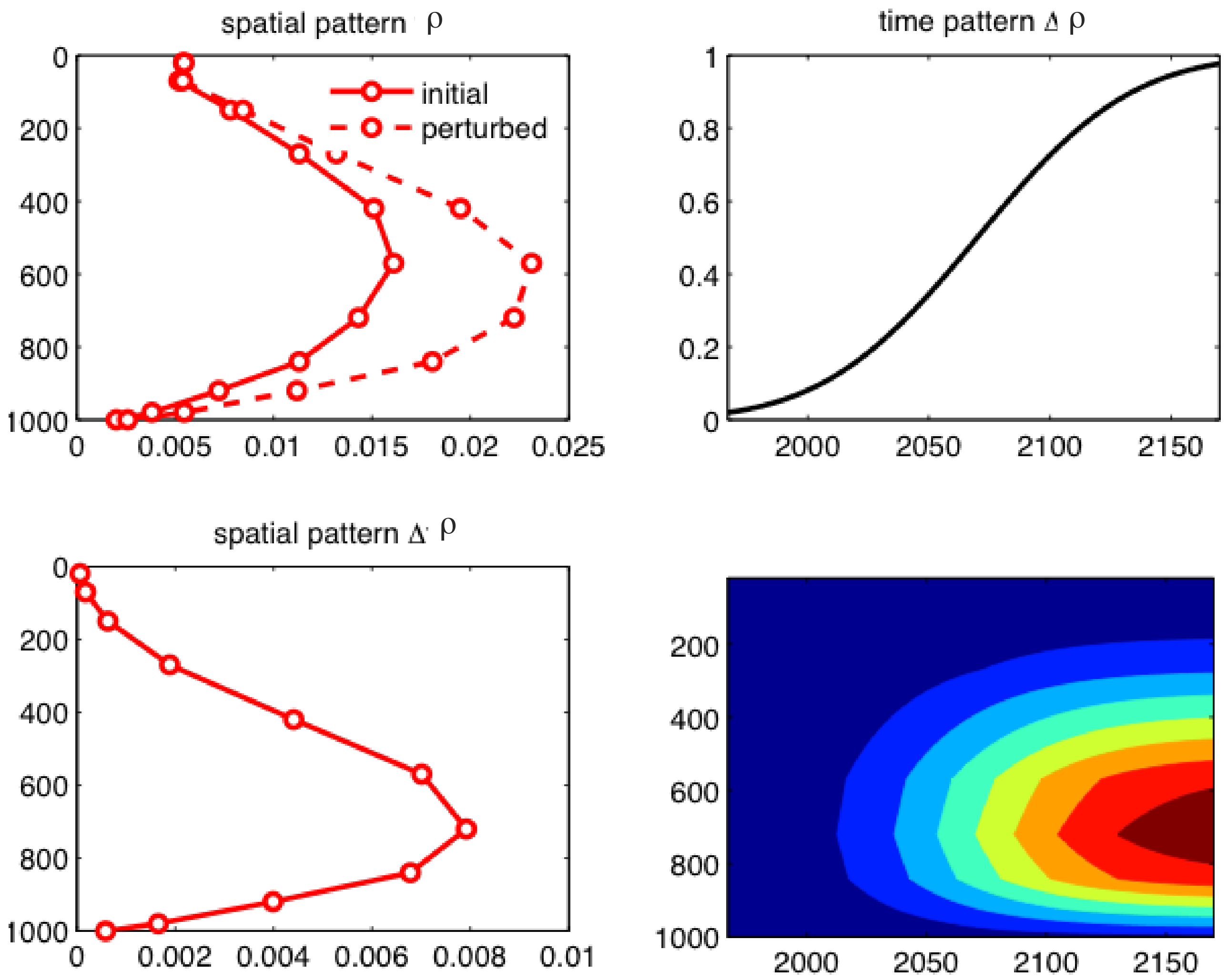


Resolution of the model

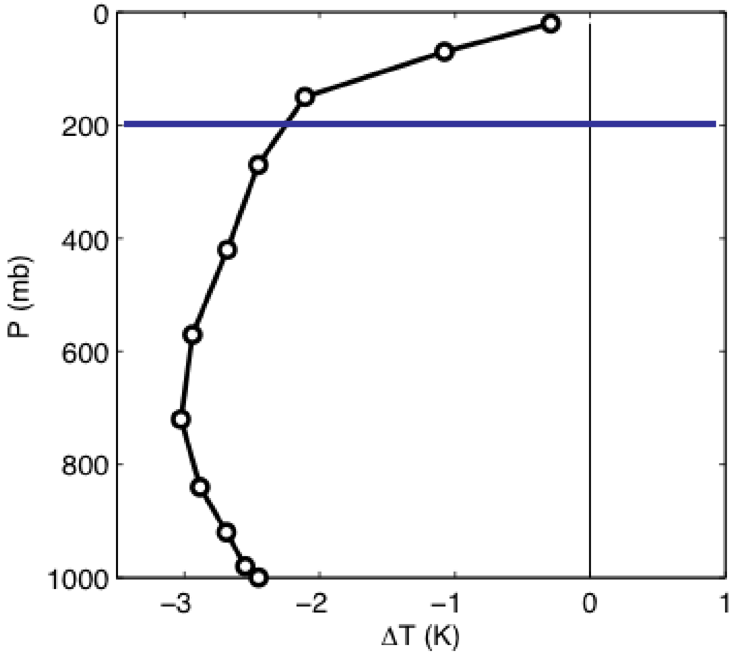
- Prescribed: time-trajectory of the spatial pattern of ρ (aerosol forcing)



Resolution of the model

- Result: time-trajectory of the spatial pattern of temperature (GHG forcing only)

1D model



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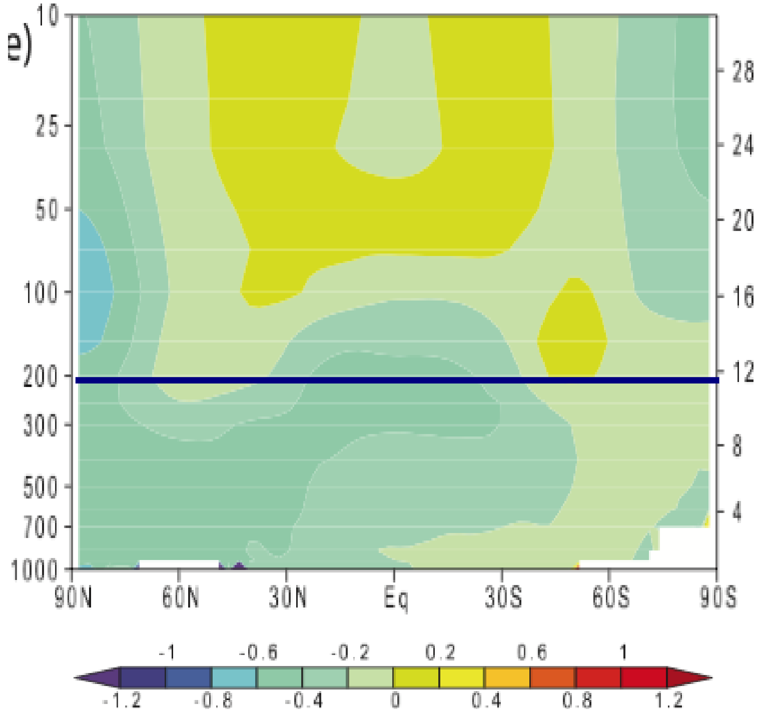
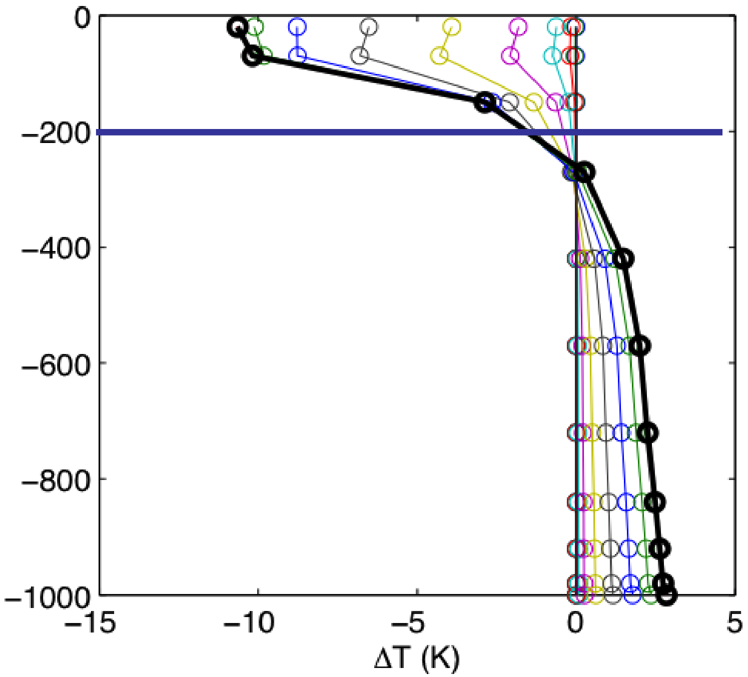


Resolution of the model

- Result: time-trajectory of the spatial pattern of temperature (all forcings)

1D model



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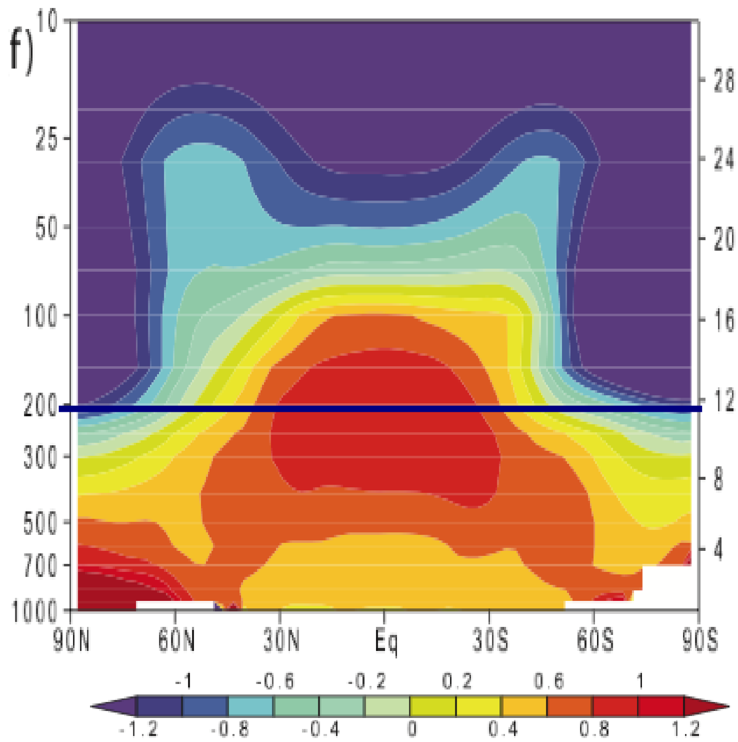


Parameterization of forcings

- Forcing *j* at time *t* and location *i*:

$$F_j(t, i; \beta_j, \mu_{,j}\, \sigma_j, m_j, s_j) = \beta_j \; \times \; f_j(t; m_j, s_j) \; \times \; g_j(i; \mu_j, \sigma_j)$$

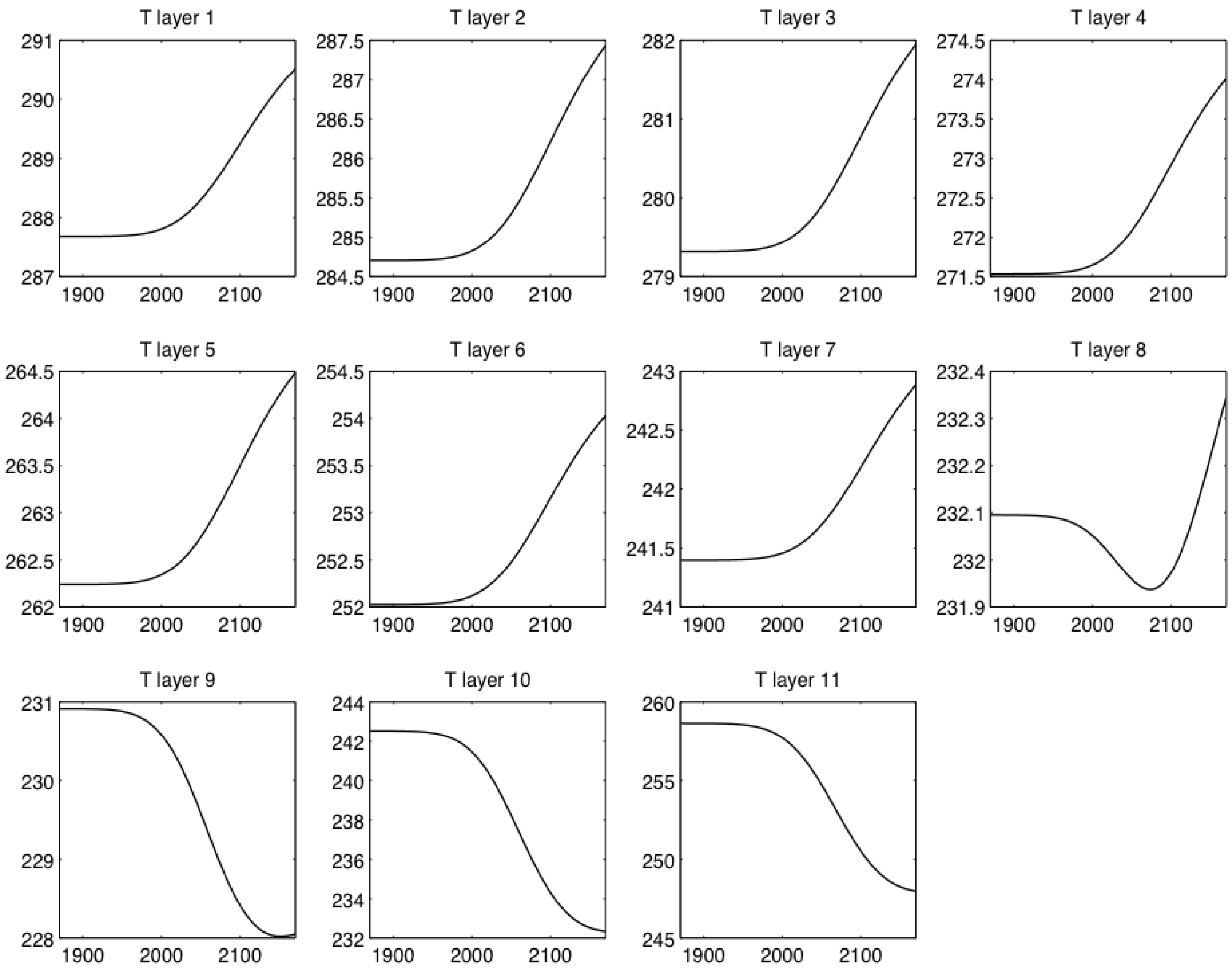
- Five scalar values for each three forcings (15 parameters):
  - magnitude β,
  - position of time pattern *m*, shape of time pattern *s*,
  - position of spatial pattern μ, shape of spatial pattern σ.

- One vector of parameters:

$$\theta = (\beta_j, \mu_j, \sigma_j, m_j, s_j)_{j=1,2,3}$$

Resolution of the model

- Result: time-trajectory of the spatial pattern of temperature (all forcings)



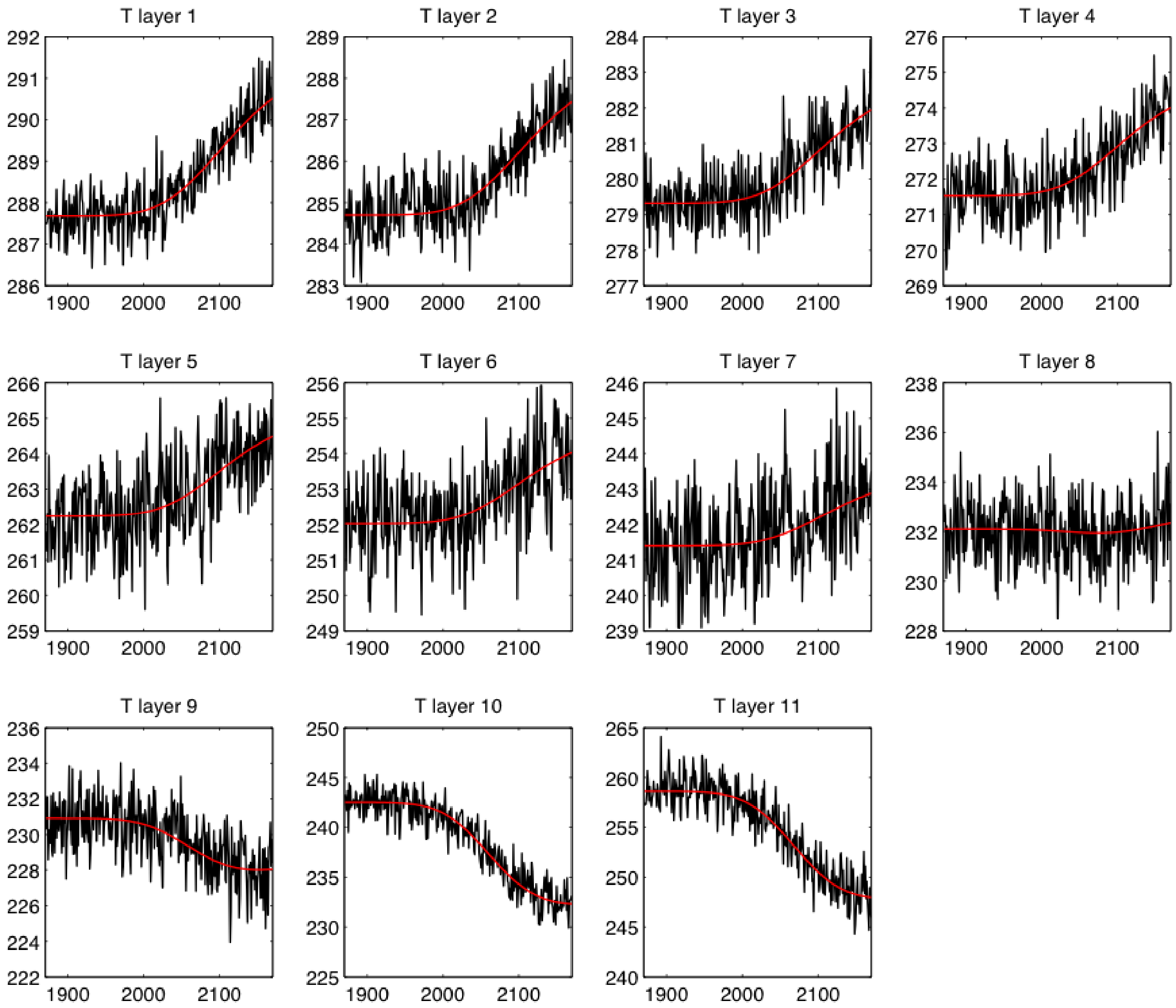
Simulation of observations

- Observed temperature:
  - model simulated temperature (deterministic)
  - internal variability (stochastic)
  - measurement error (stochastic)

$$T^{o}_{i,t} = T_{i,t} + \varepsilon_{i,t} + \eta_{i,t}$$

Simulation of observations

- Observed temperature:



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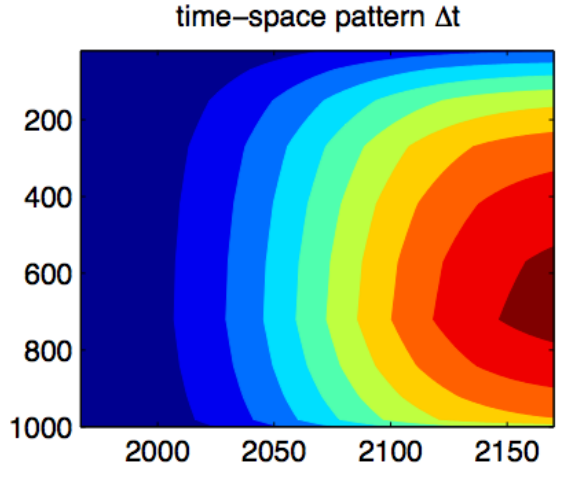
Infering forcings from observations

- Observed quantities:
  - Temperature at every time t and location i.
- Unobserved quantities to be evaluated:
  - State: climatologic temperature at every time t and location i.
  - Parameters: forcings space-time patterns + physical parameters.

$$\Rightarrow \quad F_j(t, i; \beta_j, \mu_{,j}\, \sigma_j, m_j, s_j) = \beta_j \;\times\; f_j(t; m_j, s_j) \;\times\; g_j(i; \mu_j, \sigma_j)$$



Infering forcings from observations

- Can we reconstruct unobserved radiative forcings from observed temperatures ?

- Approach: data assimilation on state variables and parameters.
  - State-augmented vector:

$$x_t = (\mathbf{T}_t, \theta)$$

  - Dynamic equation:

$$x_{t+1} = (\mathbf{T}_{t+1}, \theta) = (F(\mathbf{T}_t; \theta), \theta) = M(x_t)$$

  - Observation equation:

$$y_t^o = \mathbf{T}_t + \varepsilon_t = H(x_t) + \varepsilon_t \qquad \mathrm{Var}(\varepsilon_t) = \mathbf{R}$$

Infering forcings from observations

- Sequential inference:
  - Forecast step:

    $$p(x_t \mid I_{t-1}) \text{ is obtained from } p(x_{t-1} \mid I_{t-1}) \text{ and } x_t = M(x_{t-1})$$

  - Analyis step:

    $$p(x_t \mid I_t) = p(x_t \mid y_t^o, I_{t-1}) \propto p(y_t^o \mid x_t) \times p(x_t \mid I_{t-1})$$

- Linearization:
  - Tangent model (closed form):

    $$\mathbf{L}_t = \partial M/\partial x \mid_{x_t} = \begin{pmatrix} \partial F/\partial \mathbf{T} \mid_{\mathbf{T}_t} & \partial F/\partial \theta \mid_{\mathbf{T}_t} \\ \mathbf{0} & \mathbf{I} \end{pmatrix}$$

  - Forward operator (linear in the first place):

    $$\mathbf{H} = \partial H/\partial x = \begin{pmatrix} \mathbf{I} & \mathbf{0} \end{pmatrix}$$

Extended Kalman Filter procedure

- Extended Kalman filter:
  - Forecast step:

$$\begin{cases} x_t^f = \mathbb{E}(x_t \mid I_{t-1}) = M_{t-1}(x_{t-1}^a) \\ \mathbf{P}_t^f = \mathrm{Var}(x_t \mid I_{t-1}) = \mathbf{L}_{t-1}\mathbf{P}_{t-1}^a\mathbf{L}_{t-1}' \end{cases}$$

  - Analysis step:

$$\begin{cases} x_t^a = \mathbb{E}(x_t \mid I_t) = x_t^f + \mathbf{K}_t d_t \\ \mathbf{P}_t^a = \mathrm{Var}(x_t \mid I_t) = (\mathbf{I} - \mathbf{K}_t\mathbf{H})\,\mathbf{P}_t^f \end{cases}$$

  - Intermediate quantities:

$$\begin{cases} d_t = y_t^o - \mathbb{E}(y_t^o \mid I_{t-1}) = y_t^o - H(x_t^f) \\ \mathbf{K}_t = \mathbf{P}_t^f\mathbf{H}'\,(\mathbf{R} + \mathbf{H}\,\mathbf{P}_t^f\mathbf{H})^{-1} \end{cases}$$

Initialization

- Initializing requires an a priori distribution on x_0:

$$\pi(x_0) = \mathcal{N}(x_0^f, \mathbf{P}_0^f) \quad = ? \qquad x_0^f = ? \qquad \mathbf{P}_0^f = ?$$

- A priori independence between temperatures and parameters:

$$\pi(x_0) = \pi(\mathbf{T}_0) \times \pi(\theta)$$

- Natural choice for temperatures:

$$\pi(\mathbf{T}_0) = \mathcal{N}(\bar{\mathbf{T}}, \mathbf{R})$$

Initialization

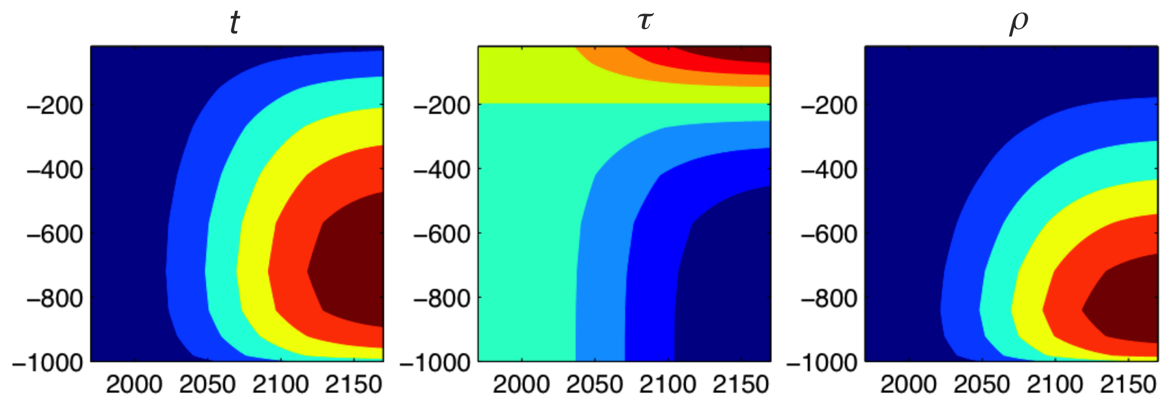
- What about prior distributions of parameters ?
  - Should it be informative or non informative ?
  - If informative, ‘objective expert’ a priori or ‘devil’s advocate’ a priori ?

$$\pi(\theta) = ?$$

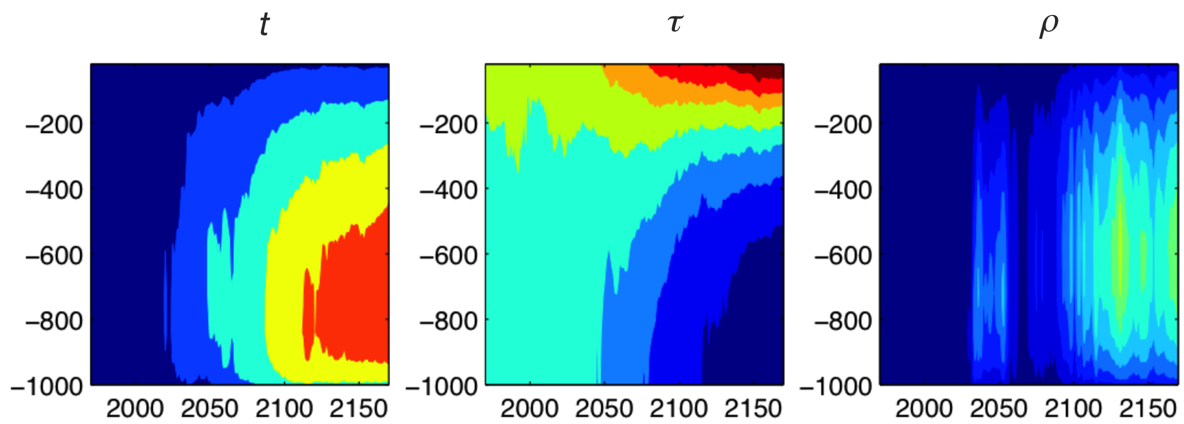
- ‘fingerprinting-like’ option chosen:
  - Non informative prior
  - e.g. $E(\beta) = 0$, $Var(\beta) = 10$

Results

- Actual space-time patterns of forcings:

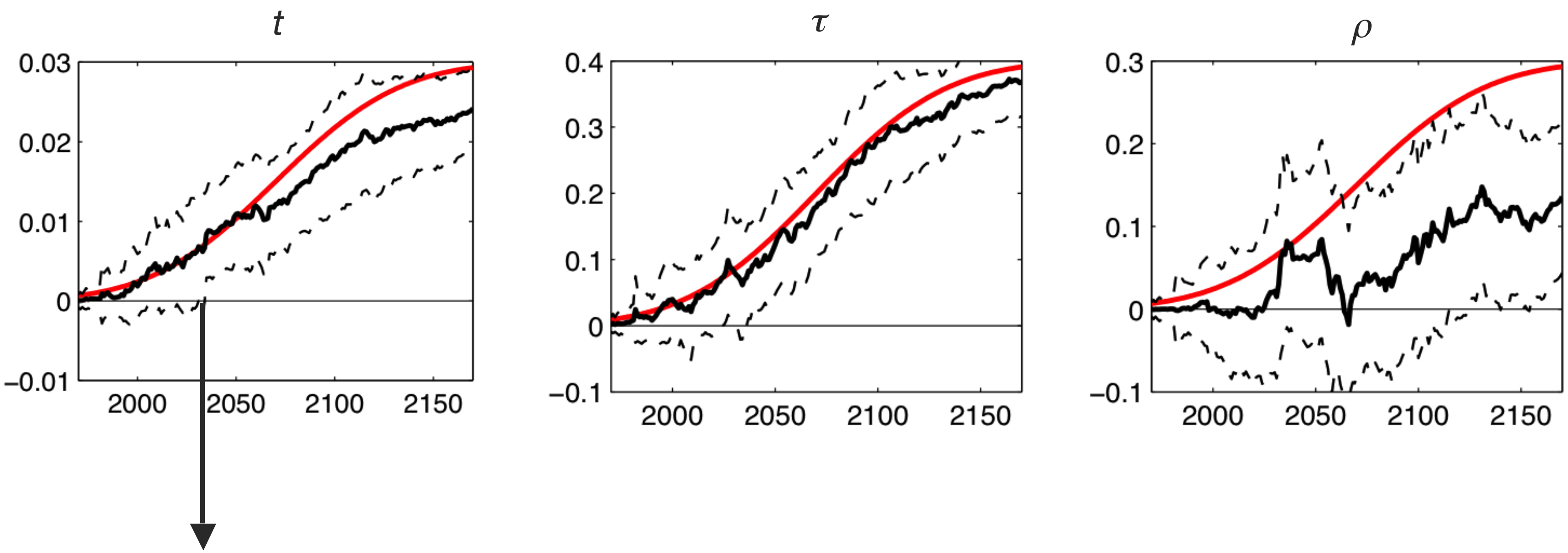


- Reconstructed space-time patterns of forcings:



Results

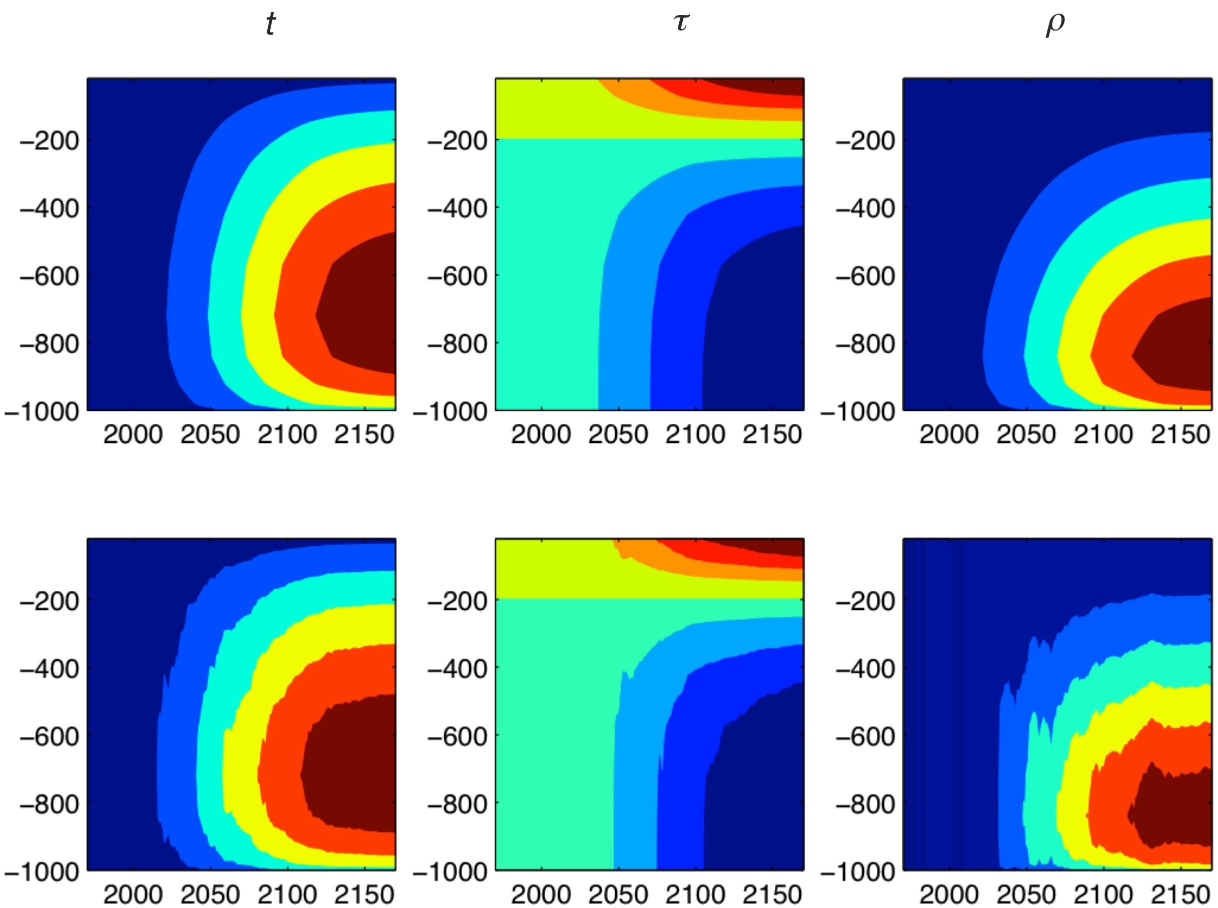
- Actual (red) and reconstructed (dark) magnitudes of forcings:



Attribution to GHG emissions in year 2035

Results

- Space-time patterns are now assumed to be known up to a scaling factor (only parameter β is not known):



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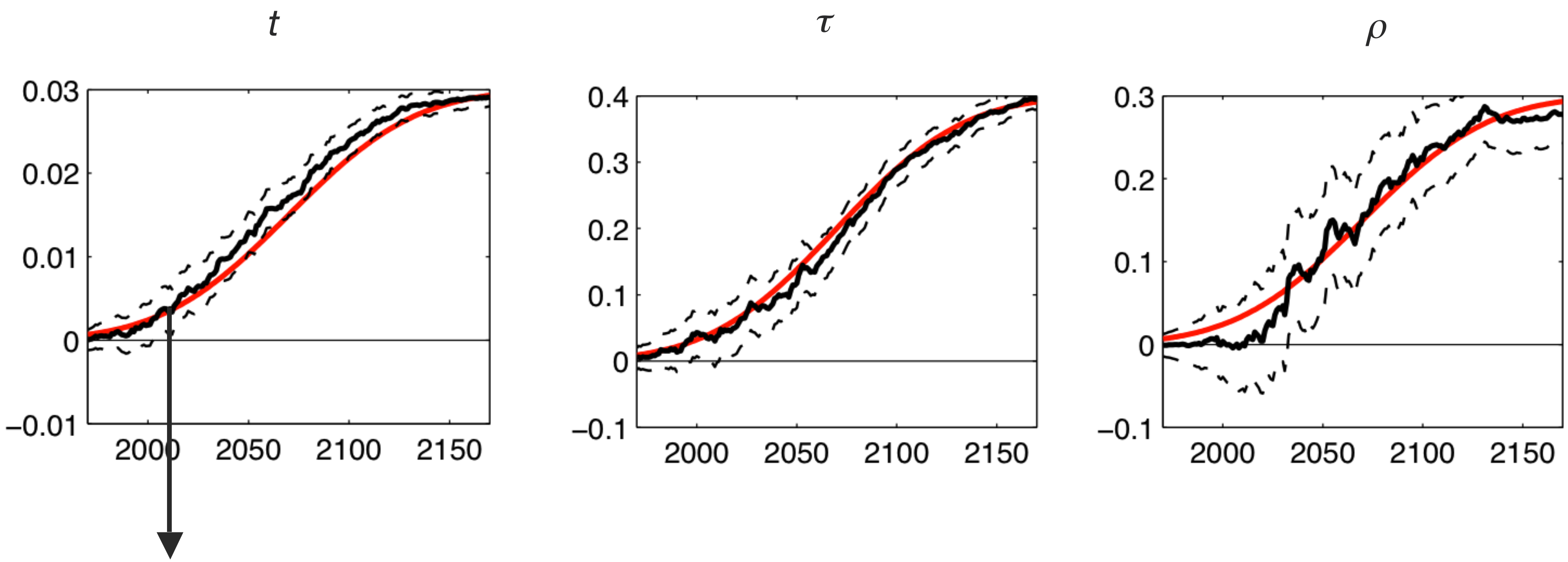

Attribution to GHG emissions in year 2010

Results

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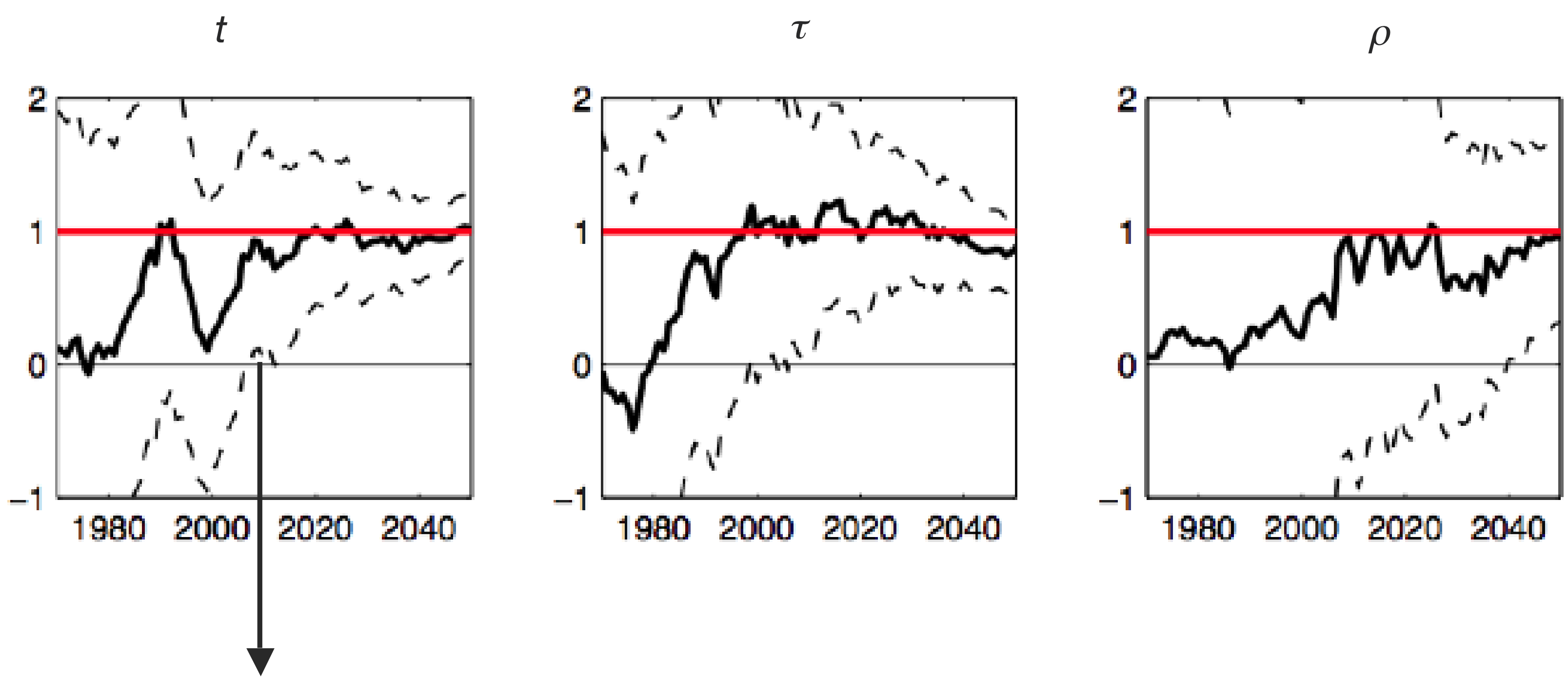

Attribution to GHG emissions in year 2010

Infering forcings from observations

- Linearization:
  - Tangent model (closed form):

$$\mathbf{L}_t = \partial M / \partial x \mid_{x_t} = \begin{pmatrix} \partial F / \partial \mathbf{T} \mid_{\mathbf{T}_t} & \partial F / \partial \theta \mid_{\mathbf{T}_t} \\ \mathbf{0} & \mathbf{I} \end{pmatrix}$$

Infering forcings from observations

$$G(\rho, t, \tau) = (\mathbf{K}^{-1} E/\sigma)^{1/4}$$

$$\mathbf{K} = \begin{cases} k_{00} = -1, \\ k_{ii} = -2(1-t_i), \quad i = 1, 2, ..., n, \\ k_{01} = (1-t_1), \\ k_{0j} = (1-t_j) \prod_{l=1}^{l=j-1} t_l, \quad j = 2, 3, ..., n, \\ k_{i,i+1} = (1-t_i)(1-t_{i+1}), \quad i = 1, 2, ..., n, \\ k_{ij} = (1-t_i)(1-t_j) \prod_{l=i+1}^{l=j-1} t_l, \quad i = 1, 2, ..., n \ (j > i+1) \end{cases}$$

$$E = \begin{cases} E_0 = S/4(1-\rho_0) \prod_{l=1}^{l=n} \tau_l, \\ E_i = S/4(\prod_{l=i+1}^{l=n} \tau_l)\{1 - \tau_i - \rho_i + \tau_i(1-\tau_i)[\sum_{l=0}^{l=i-1} (\rho_l \prod_{m=l+1}^{m=i-1} \tau_m^2)]\} \end{cases}$$

Statistical ‘evidencing’ for D&A in EKF

- Can be approached as a comparison between two priors
  - An objective a priori on forcings (actual GHG level)
  - A devil’s advocate a priori (no GHG for instance)

- The Bayes factor is an option:
  - Informational metric
  - Exact expression available in the EKF context

$$B(\pi^{(0)}, \pi^{(1)}) = \frac{m^{(0)}(\mathbf{y}^o)}{m^{(1)}(\mathbf{y}^o)}$$

$$m^{(k)}(\mathbf{y}^o) = \int_{\mathbf{x},\theta} p(\mathbf{y}^o \mid \mathbf{x}, \theta) \,.\, \pi^{(k)}(\mathbf{x}, \theta) \,.\, \mathrm{d}\mathbf{x} \,.\, \mathrm{d}\theta$$

Jeffrey's scale

An absolute scale is available:

K	dB	bits	Strength of evidence
1:1 to 3:1	0 to 5	0 to 1.6	Barely worth mentioning
3:1 to 10:1	5 to 10	1.6 to 3.3	Substantial
10:1 to 30:1	10 to 15	3.3 to 5.0	Strong
30:1 to 100:1	15 to 20	5.0 to 6.6	Very strong
> 100:1	> 20	> 6.6	Decisive

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Inclusion of confounding factors

- The inference procedure is run with:
  - inclusion of two extra parameters representing albedo and solar change.

$$\theta = \{(\beta_j, \mu_j, \sigma_j, m_j, s_j)_{j=1,2,3}, S, \rho_0\}$$

  - strongly informative a priori that GHG forcing is inexistant.
  - non informative a priori about solar and albedo changes.
  - observations simulated with a GHG-only forcing.

Inclusion of confounding factors

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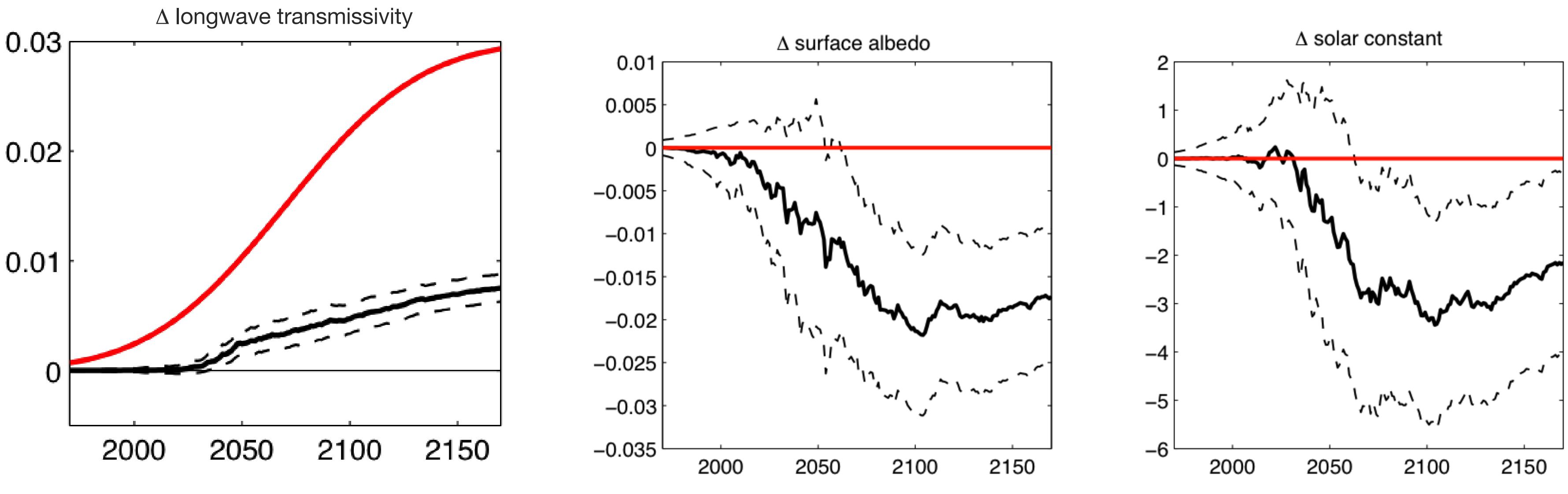
  - inclusion of two extra parameters representing albedo and solar change.
  - strongly informative a priori that GHG forcing is inexistant.
  - non informative a priori about solar and albedo changes.
  - observations simulated with a GHG-only forcing.

- Results:
  - Wrong attribution to solar constant and albedo change.
  - Substantial underestimation of GHG magnitude.



Inclusion of confounding factors

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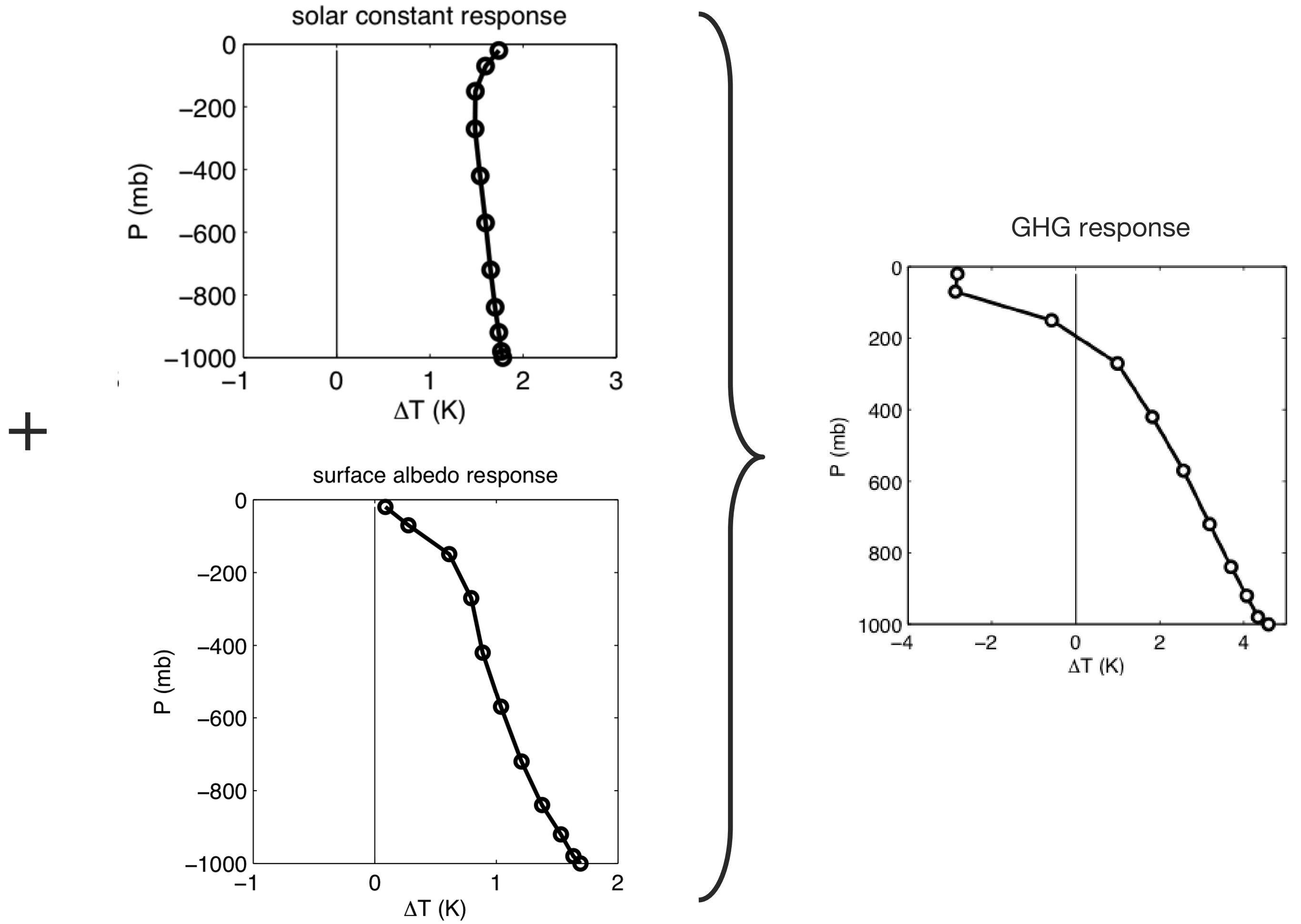
  - inclusion of two extra parameters representing albedo and solar change.
  - strongly informative a priori that solar and albedo changes are inexistant.
  - non-informative a priori about GHG change.
  - observations simulated with solar and albedo change forcing only (no GHG).

- Results:
  - Wrong attribution to GHG change.

Inclusion of confounding factors

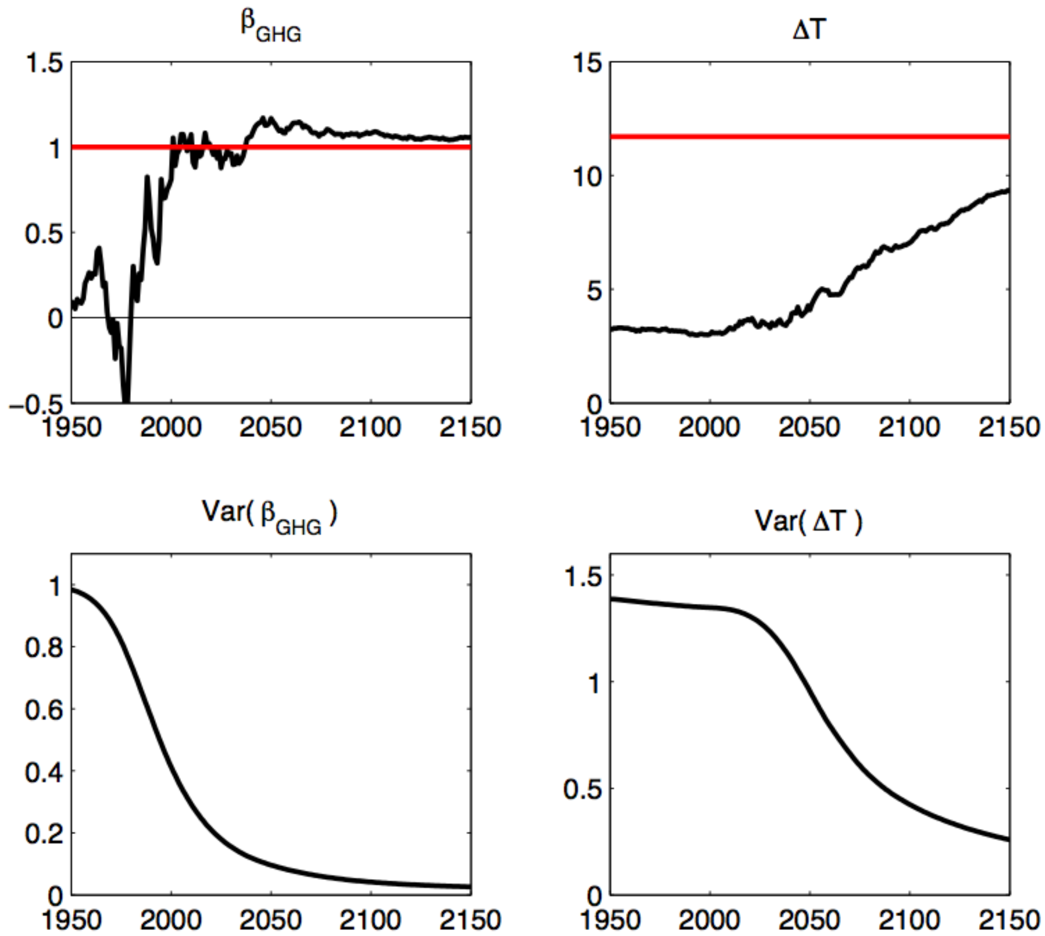
- The combination of a decrease in solar constant with a decrease in surface albedo results in a pattern identical to an increase in GHG.



Inclusion of uncertain physical quantities

- The framework can be extended for simultaneous inference of other parameters representing uncertain climatic quantities of interest.

- Climate sensitivity is assumed uncertain (due to feedback):



Limitation of EKF: overconfidence

- Kalman filtering of constant parameters can only decrease uncertainty.

- This is a big limitation of KF for inference of constant parameters:
  - decrease of variance is actually far from being systematic
  - typically, variance increases result from 'surprising' observations

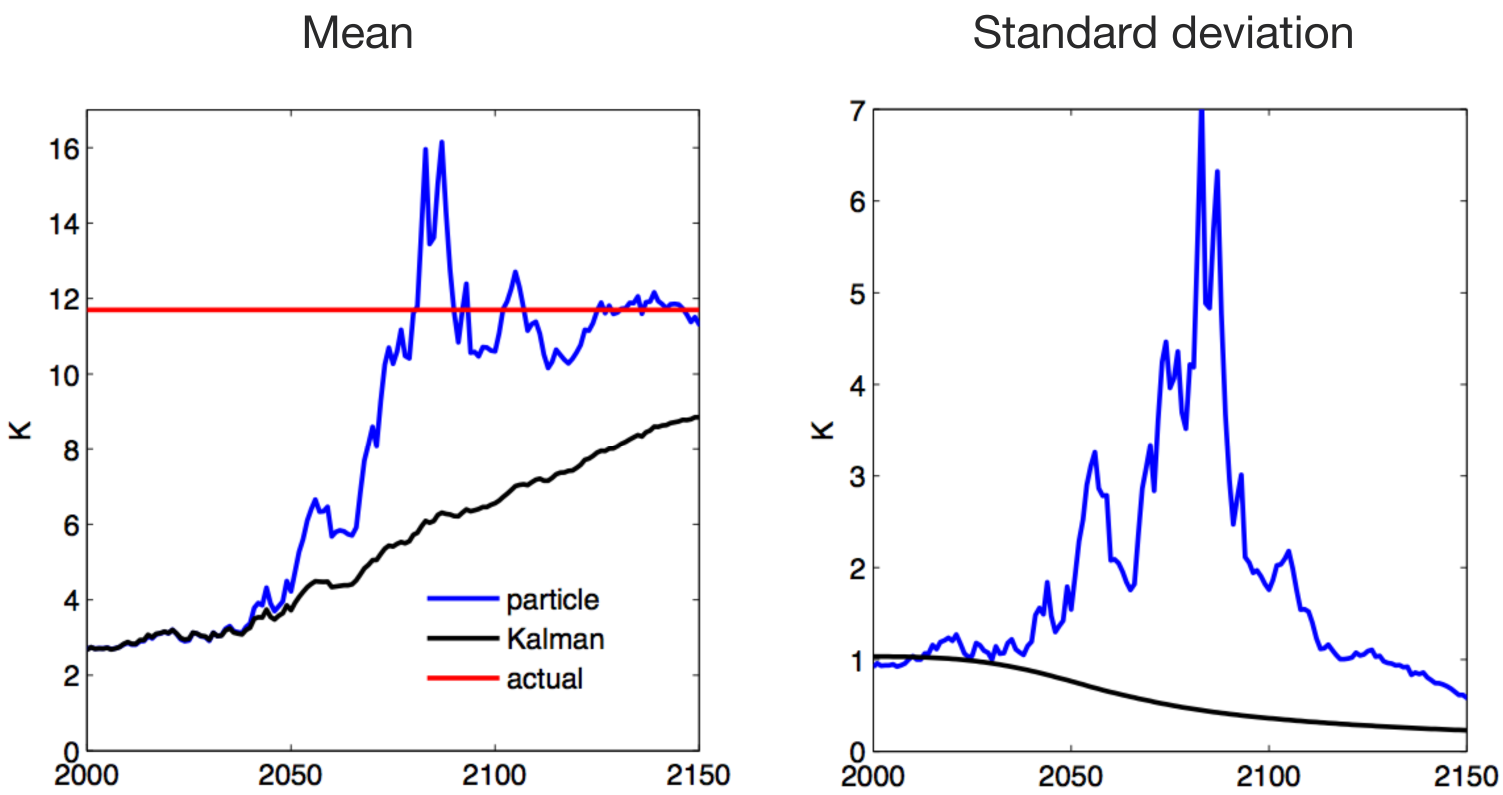
$$\text{Let:} \begin{cases} \mathbf{P} = \begin{pmatrix} \mathbf{P}_{xx} & \mathbf{P}_{x\mu} \\ \mathbf{P}'_{x\mu} & \mathbf{P}_{\mu\mu} \end{pmatrix}, \ \mathbf{L} = \begin{pmatrix} \mathbf{L}_x & \mathbf{L}_\mu \\ \mathbf{0} & \mathbf{I} \end{pmatrix}, \ \mathbf{H} = \begin{pmatrix} \mathbf{H}_x & \mathbf{0} \end{pmatrix} \\ \mathbf{\Delta}_{x\mu} = \mathbf{L}_x \mathbf{P}^a_{x\mu} + \mathbf{L}_\mu \mathbf{P}^a_{\mu\mu}, \quad \mathbf{J}_x = \mathbf{H}'_x (\mathbf{R} + \mathbf{H}_x \mathbf{P}^f_{xx} \mathbf{H}'_x)^{-1} \mathbf{H}_x \end{cases}$$

$$\Rightarrow \mathbf{P}^a_{\mu\mu,t} = \mathbf{P}^a_{\mu\mu,t-1} - \mathbf{\Delta}'_{x\mu,t-1} \mathbf{J}_{x,t} \mathbf{\Delta}_{x\mu,t-1}$$

$$\Rightarrow (\mathbf{P}^a_{\mu\mu,t})_{ii} \leq (\mathbf{P}^a_{\mu\mu,t-1})_{ii} \quad \forall i \in \{1, \dots, p\}$$

Limitation of EKF: overconfidence

- Comparison to a full Bayesian inversion of climate sensitivity:



Outline of the presentation

- Motivation and purpose
- Model description
- Inversion
- Confounding factors
- Optimal fingerprinting
- Metamodeling
- Conclusions

Comparison with optimal fingerprinting

- Assuming patterns are known up to a scaling factor (parameter β) allows for direct comparison to an optimal fingerprinting treatment of inference.

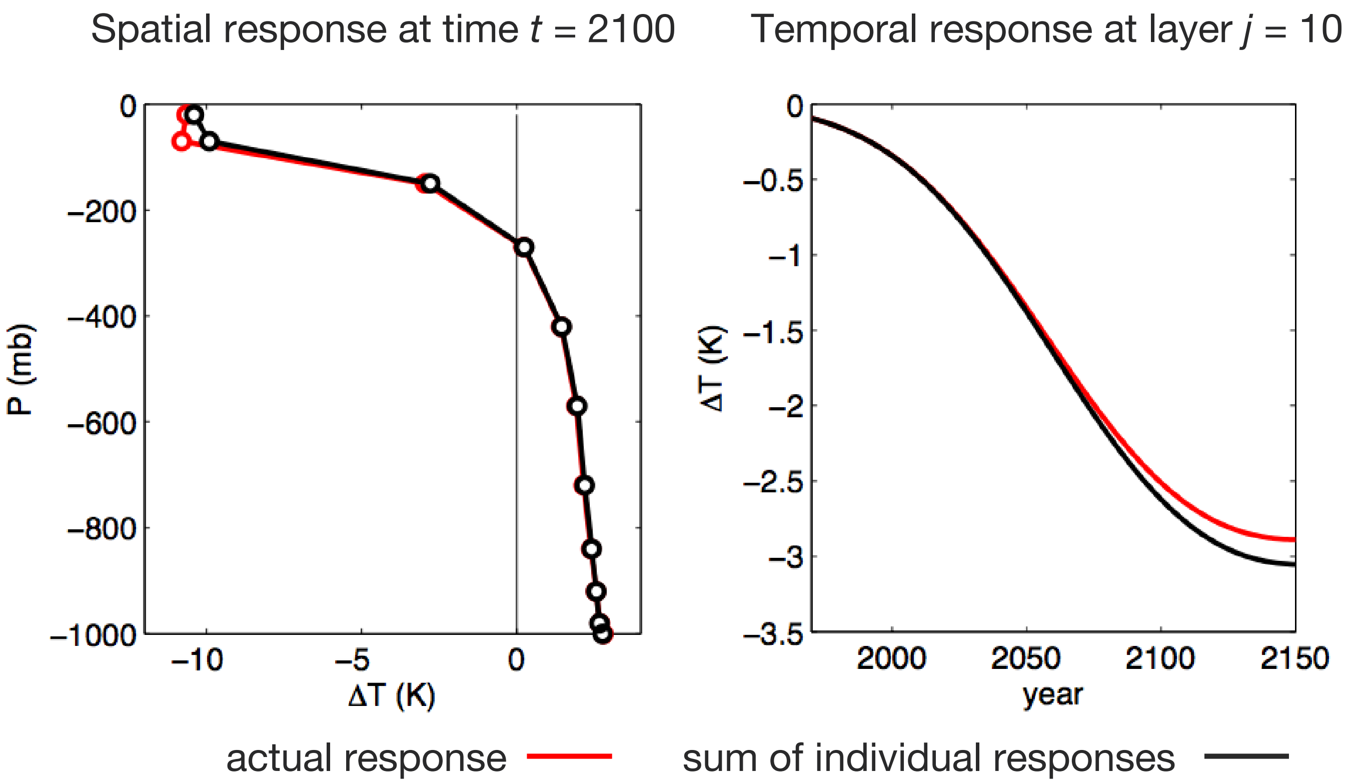
- Optimal fingerprinting setting:
  - Generation of the single-forcing model simulations, $z_t^{(j)}$ for $j = 1, \ldots, J$.
  - Concatenation of all variables up to time t
  - Linear regression model - Total Least Squares (Allen and Stott [2003]):

$$Y_t^o = (\mathbf{Z}_t - \boldsymbol{\nu}_t)\beta + \eta_t \qquad \left\{ \begin{array}{l} Y_t^o = (y_0^o, y_1^o, \ldots, y_t^o) \\ Z_t^{(j)} = (z_0^{(j)}, z_1^{(j)}, \ldots, z_t^{(j)}) \\ \mathbf{Z}_t = (Z_t^{(1)}, Z_t^{(2)}, \ldots, Z_t^{(J)}) \end{array} \right.$$

where $\mathrm{Var}(\nu) = \mathrm{Var}(\eta)/n_r$ and n_r is the number of simulated runs.

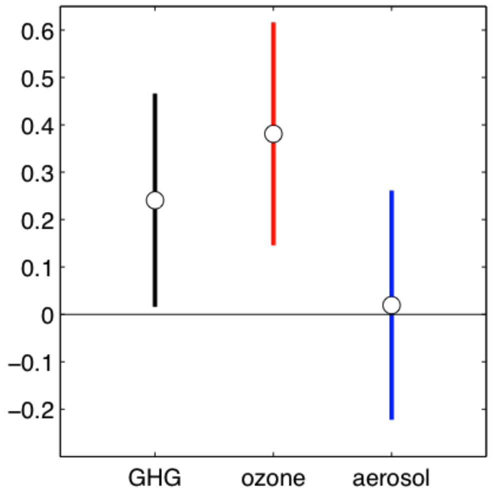
Comparison with optimal fingerprinting

- The assumption of response additivity holds very well both spatially and temporally:



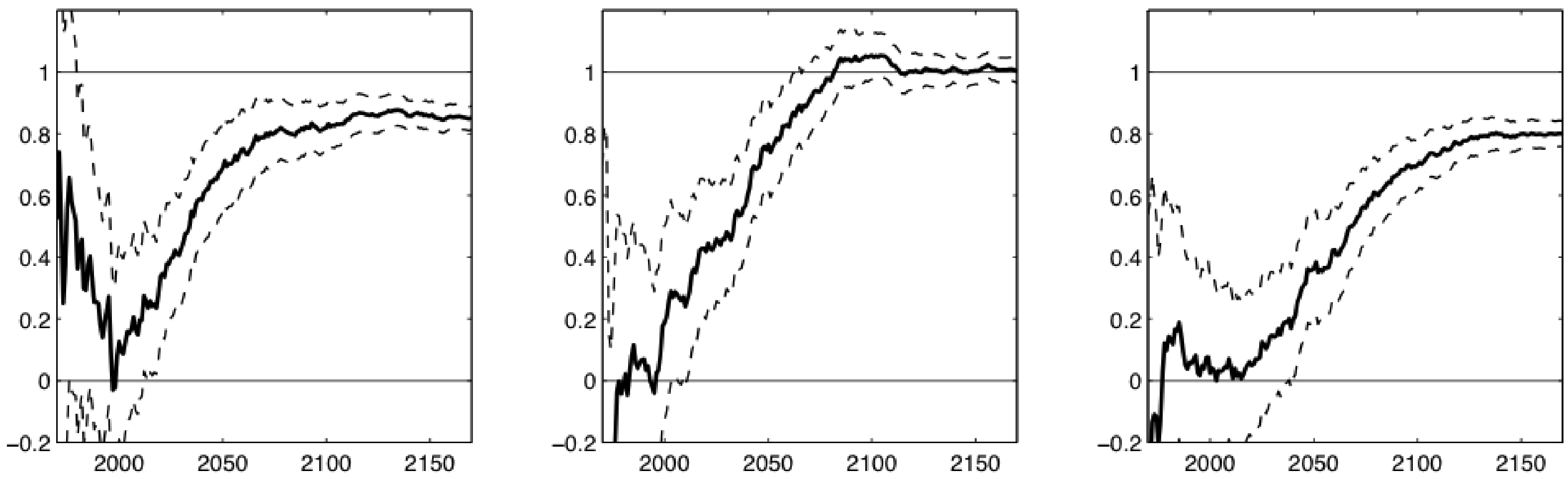
Comparison with optimal fingerprinting

- Scaling parameters β at time t = 2015:



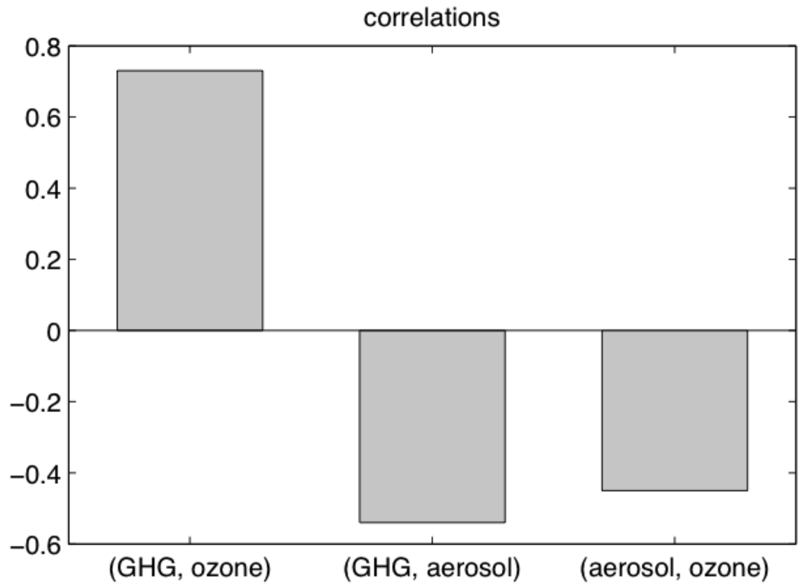
Attribution to GHG emissions in year 2015

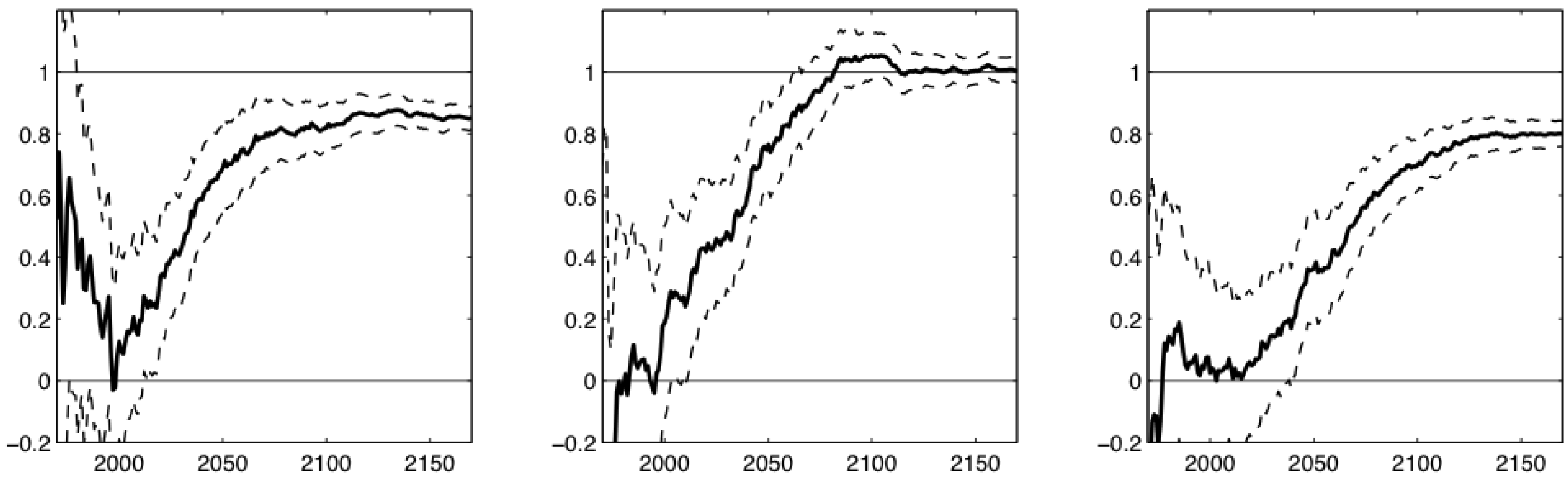
- Scaling parameters β at any time *t*:



Comparison with optimal fingerprinting

- Non convergence of factors β (non orthogonal patterns ?)





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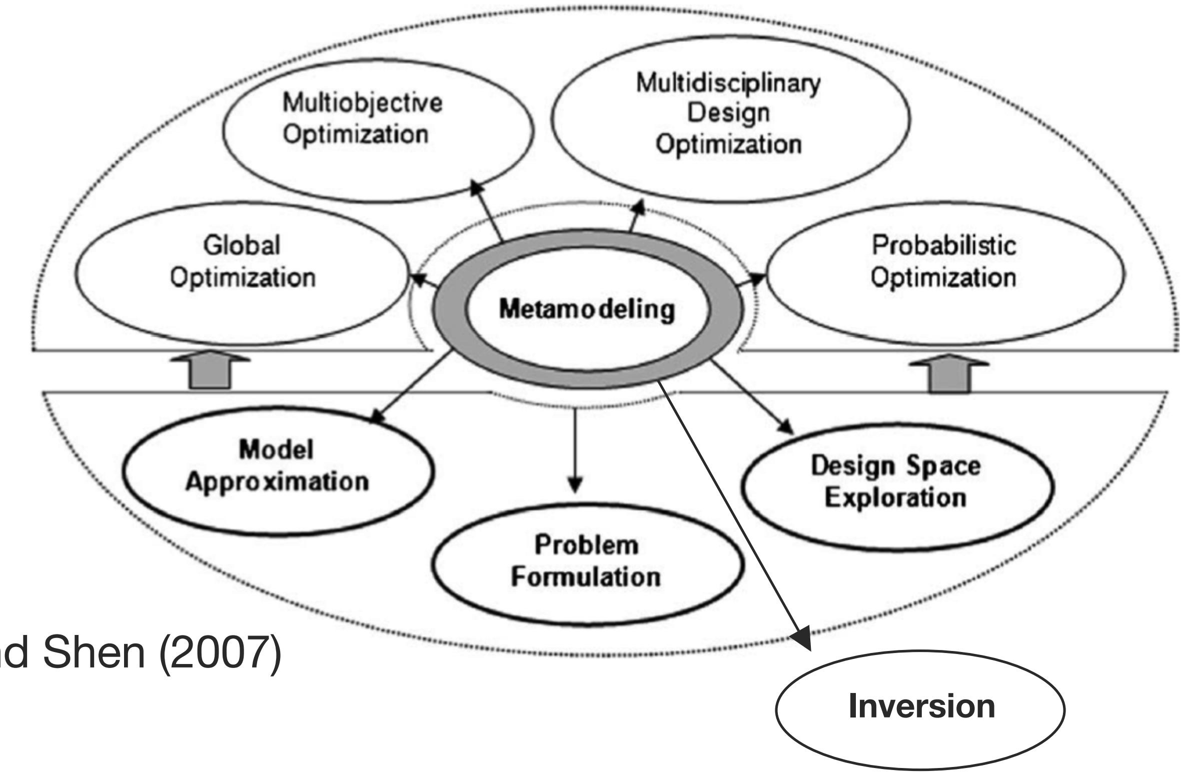
Metamodeling

- Notion of ‘metamodel’.
  - simplified, parametric model of a large, complex model.
  - physical model, statistical model, or both.
  - used in different fields (oil industry, aeronautics, …)
  - other names in climate science: ‘emulator’, ‘response function’,…

- Multiple purposes:



Wang and Shen (2007)

Metamodeling

- Optimal fingerprinting can be formulated under a DA set-up:
  - it can be resolved 'step by step' using the EKF,
  - rather than 'all at a time' using TLS.

$$Y_t^o = (\mathbf{Z}_t - \boldsymbol{\nu}_t)\beta + \eta_t \qquad \text{with:} \quad \begin{cases} Y_t^o = (y_0^o, y_1^o, \dots, y_t^o) \\ Z_t^{(j)} = (z_0^{(j)}, z_1^{(j)}, \dots, z_t^{(j)}) \\ \mathbf{Z}_t = (Z_t^{(1)}, Z_t^{(2)}, \dots, Z_t^{(J)}) \end{cases}$$

$$\Leftrightarrow$$

$$\begin{aligned} x_{t+1} &= M(x_t) + \nu_t \\ y_t^o &= H(x_t) + \eta_t \end{aligned} \qquad \text{with:} \quad \begin{cases} M(x_t) = \mathbf{z}_{t+1}\beta \\ \mathbf{z}_t = (z_t^{(1)}, z_t^{(2)}, \dots, z_t^{(J)}) \end{cases}$$

Metamodeling

- Optimal fingerprinting can be viewed as a particular kind of metamodel:
  - purely statistical metamodel.
  - no explicit physics (entirely summarized by runs).

$$M(x_t) = \mathbf{z}_{t+1}\beta$$

- Many possible other metamodels for D&A
  - purely statistical (EMR)
  - statistical and physical (fitting a toy model to a GCM)

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Discussion and conclusion

- Results of the DA-based inversion of the 1D model:
  - space-time patterns of forcing perturbation decently reconstructed.
  - attribution can be derived from reconstructions' confidence intervals.
  - the computational constrain is still far away (~10s for 300 years of DA).

- Results highlight some avantages of DA-based inversion:
  - specification of fingerprints beforehand is unnecessary.
  - allows for precise modeling of the initial state of knowledge on the perturbation (e.g. spatial and temporal patterns).
  - easy inclusion of confounding effects, even 'unknown'.
  - easy inclusion of uncertainty on poorly known physical parameters.

Discussion and conclusion

- More avantages of a DA-based inversion approach:
  - handling of model error.
  - handling of measurement error.
  - incorporating multiple sources of observations (variable and forcings).
  - handling of multiple state variables and observations simultaneously.
  - coupling with impact model and observations.
  - straightforward derivation of Bayes factor.

- Metamodeling:
  - metamodeling can be a workaround to use complex models for D&A in much less computational time.
  - optimal fingerprinting can be viewed as a particular case of metamodel.

- Possible extension to a more general « climate learning » framework.
  - Observations constrain both forcings and model's parameters.

Discussion and conclusion

- Difficulties of the DA-based approach:
  - best strategy to handle time-varying forcings unclear (determinist pattern parameterization or stochastic model ?).
  - the variance decrease on fixed parameters is an issue (use particle filter instead ?).
  - potentially a long list of technical problems to solve if we want to ‘go big’.