Optimization of trading policies for wind energy producer

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Los ignorantes suponen que infinitos sorteos requieren un tiempo infinito; en realidad basta que el tiempo sea infinitamente subdivisible.

$* * *$

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(Kolmogorov's work laying the foundations of the theory of continuous time stochastic processes was only published in 1933)

1. Introduction
2. A model for realized production
3. A model for forecast dynamics
4. Optimization of market interventions

Context: wind energy in France

- As of 2014, the total installed wind power capacity in France was 9,285 MW, 3.1% of total electricity was produced from wind.
- The market is not dominated by a single producer: 1200 MW is installed by Engie (GDF Suez), 850 MW by EDF Energies Nouvelles, the rest is split between many independent producers.

Context: wind energy in France

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- The market is not dominated by a single producer: 1200 MW is installed by Engie (GDF Suez), 850 MW by EDF Energies Nouvelles, the rest is split between many independent producers.
- Presently, the production of a wind power plant is bought by EDF at a fixed price for the first 15 years of the plant's operation.
- After the "guaranteed purchase" period, the producers must sell the electricity in the open market

Electricity markets in Europe

Wind power producers have access to four types of markets:

- Forward market – up to 1 day prior to delivery
- Spot market – 1 day prior to delivery
- Intraday market – between 1 day and 45 minutes
- Adjustment (imbalance) market (managed by RTE, power network operator) – the last 45 minutes

Forward market | Spot market | Intraday market | Adjustment market

$t-24h$ | $t-45min$ | Production date t

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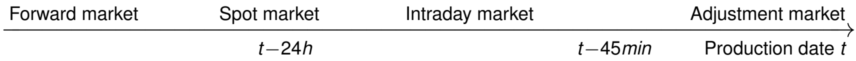

Short-term forecasts may be used to determine optimal trading strategies

Goal and contributions of this study

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- Compare the gain of the wind producer with/without forecast, to determine the economic value of the forecasts.

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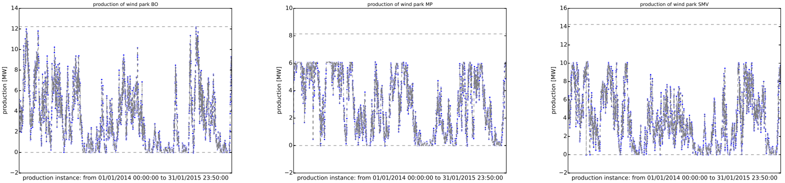
Main contributions

- We propose a stochastic dynamic model for the realized production and the forecast errors and calibrate it to data provided by a wind producer.
- We formulate the optimization problem faced by the wind producer and determine the optimal solution under various assumptions.

Production data

- Output power at wind park level for 3 wind parks in France, sampled at 10-minute intervals from Jan 1st, 2011 to Jan 1st, 2015 was provided by Maïa Eolis
- Data contains small negative values due to turbine consumption
- Production caps at the rated power of wind park P_{max}



Realized production for the three wind parks in January 2014

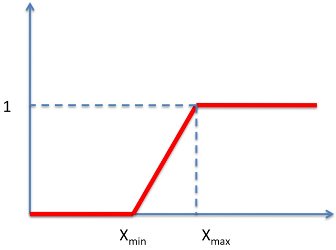
A model for realized production

The "normalized production" F_T is a function of the "stylized wind speed" X_T

$$F_T = f_{prod}(X_T), \qquad F_T = \begin{cases} \frac{P_T}{P_{max}} & 0 < P_T < P_{max} \\ 0 & P_T \leq 0 \\ 1 & P_T = P_{max} \end{cases}$$

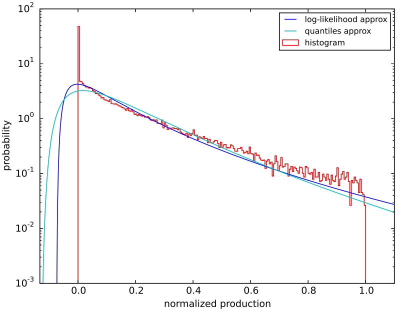
$$f_{prod}(x) = \frac{(x - x_{min})^+ - (x - x_{max})^+}{x_{max} - x_{min}}$$

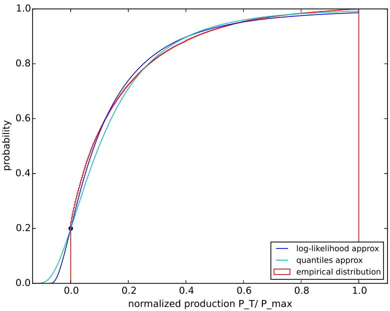
- P_T is the actual production
- P_{max} is the total rated power
- X_T is a latent variable, "stylized wind speed"
- f_{prod} is the production function (power curve)



Production: results for BO power plant

We assume that X_T follows a 2-parameter log-normal distributon. Then, $F_T \in [0, 1]$ follows a 3-parameter truncated log-normal distribution.





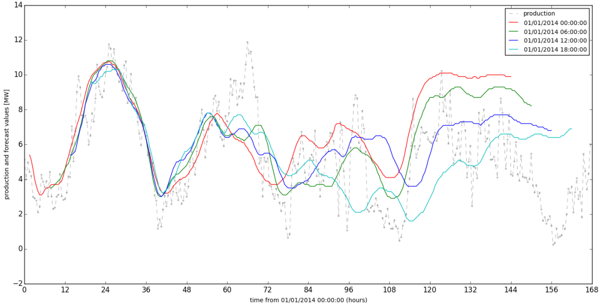
Fitted production density (left) and distribution function (right)

Forecast data

Forecasts of the output power at wind park level, produced by an independent forecasting company, were provided by Maïa Eolis

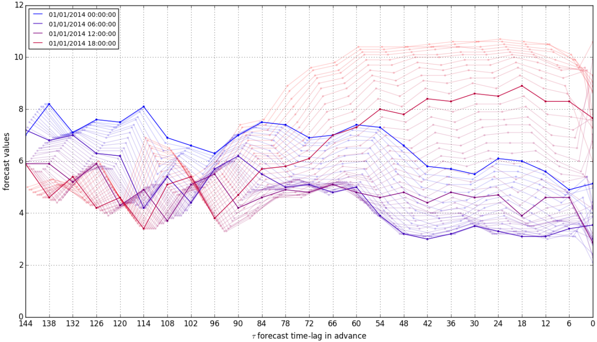
- **Period**: Dec 7th 2011 – March 3rd, 2015
- **Frequency of forecast updates**: 6 hours
- **Forecast time horizon**: from 1h15min to 144 hours with 15 minute step
- The forecast values are positive
- In the analysis, we use the normalized forecast value $F_t(T) = forecast(t, T)/P_{max}$

Forecast evolution



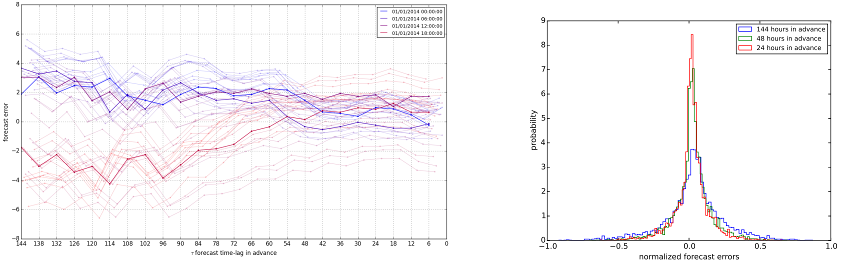
Plot of the forecast made at a given date as function of time horizon together with the realized production for this horizon. Accuracy decreases for longer horizons.

Forecast dynamics



Dynamics of the forecast made for a given delivery date. The forecast process appears continuous. Volatility increases slightly as delivery date approaches.

Forecast Error



Dynamics of the forecast error $Err_\tau(T) = F_T - F_{T-\tau}(T)$ made for a given delivery date, as a function of forecast time-lag τ. Volatility of error decreases as delivery date approaches.

A dynamic model for forecast evolution

The realized production F_T is a function of the latent variable X_T:
$F_T = f_{prod}(X_T)$

$\Rightarrow$ It is natural to assume that the forecast $F_t(T)$ will depend on the best prediction of X_T available at time t, denoted by X_t

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$\Rightarrow$ To build a model for the forecast, we endow the variable X with a log-normal martingale stochastic dynamics

$$dX_t = X_t \sigma_t dW_t \quad t \in [0, T), \quad X_T = X_{T^-} e^{bZ - \frac{b^2}{2}}, \quad Z \sim N(0, 1).$$

where W is a Brownian motion (continuous time stochastic process with independent Gaussian increments) and (σ_t) describes the evolution of the forecast error.

A dynamic model for forecast evolution

In other words,

$$X_T = X_t e^{\sqrt{\theta_t}N - \frac{\theta_t}{2}}, \quad N \sim N(0,1), \quad \theta_t = \int_t^T \sigma_s^2 ds + b^2.$$

This ensures that X_t is the best prediction of X_T given X_t: $X_t = \mathbb{E}[X_T | X_t]$.

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We then assume that the forecast is the best prediction of the realized production given the available information:

$$F_t(T) = \mathbb{E}\left[f_{prod}(X_T)|X_t\right] \quad \Rightarrow \quad F_t(T) = g(X_t, \theta_t), \quad t < T$$

where the function g has an explicit form.

The model fully describes the evolution of the forecast dynamics, while ensuring that $F_t(T) \in [0,1]$ for all t.

Parameterization of the model

The model for forecast dynamics is determined by the function (θ_t) which is roughly proportional to the variance of the forecast error $F_t(T) - F_T$.

We use a parametric volatility function $t \mapsto \sigma_t$:

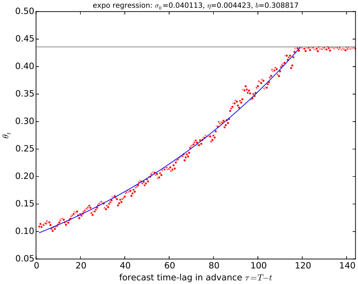
$$\sigma_t = \sigma_0 e^{\eta(T-t)} \mathbf{1}_{t > T - \tau^*}, \quad \theta_t = \frac{\sigma_0^2}{2\eta} \left(e^{2\eta(T-t)} - 1 \right) \mathbf{1}_{t > T - \tau^*} + b^2$$

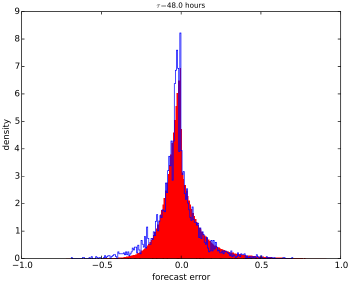
where τ^* is the stopping time of the constant volatility.

The parameters are estimated using the method of moments by fitting the empirical standard deviations of the forecast errors to those produced by the model.

Estimating the volatility function

Non-parametric estimation of θ along with the fitted parametric curve.





Left: estimated volatility function θ. More than $\tau^* = 120h$ prior to production date the forecast has no value. Right: empirical vs. model-generated density of the forecast errors for 48h time horizon.

Framework

- We focus on the gain from selling electricity during a short time period $[T - \delta, T]$, where T is fixed.
- This electricity must be sold in advance, in different markets (spot, forward, intraday), otherwise a penalty is applied for using the adjustment market.
- Our aim: determine the optimal strategy of selling electricity for a wind power producer who does not know the exact production but has forecasts available.

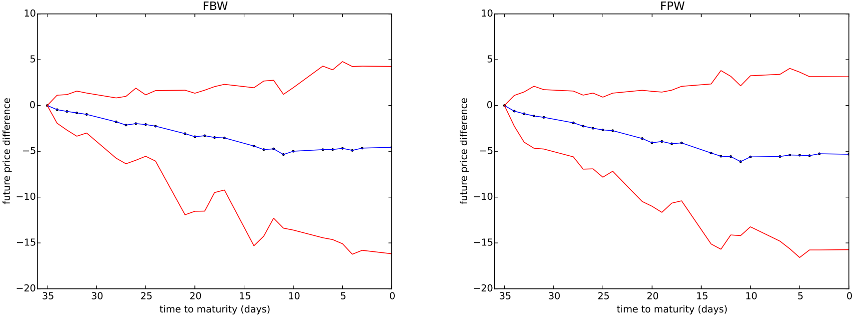
Market model

- Let $P_t(T)$ denote the forward price of one unit of electricity delivered at time T, observed at time t.
- Denote by ϕ_t the aggregate position at time t (total quantity to deliver at time T).
- We assume that ϕ is increasing process with $\phi_0 = 0$ (only sales are allowed), and that the trading starts at date 0.
- If, at date T, $\phi_T \neq F_T$, the agend must sell / purchase the extra energy at price $P_T := P_T(T)$, and in addition pay a penalty equal to $u(F_T - \phi_T)$, where $u(0) = 0$, $u(x)$ is increasing for $x > 0$ and decreasing for $x < 0$.

Why trade in different markets?

- There is insufficient market depth in the intraday market (or in other words the market impact is so strong that one can only sell large amounts of energy at a very low price).
- It is advantageous to sell in the forward market because the sale price in the spot / intraday market is lower.
- By selling in the forward market, one reduces the risk associated to the change in the price until the delivery date, since forward prices fluctuate less than spot / intraday prices.
- On the other hand, selling in the spot/intraday market reduces the penalty applied for not delivering the right amount.

Future price dynamics



Evolution of future price difference $P_t(T) - P_0(T)$ as function of t, averaged over one year, with 95% confidence bounds. Left: base futures. Right: peak futures.

Mathematical problem formulation

We assume that the forward price process satisfies

$$P_t(T) = \int_0^t \mu_s ds + \beta_t dW_t,$$

where μ and β are deterministic processes with μ_t typically negative.

Aim: maximize expected gain penalized by market impact

The problem is only affected by the randomness of the forecast

$$\min_{\psi \geq 0} \mathbb{E}\left[\underbrace{\int_0^T \phi_t \mu_t dt}_{\substack{\text{Expected loss from} \\ \text{trading in forward market}}} + \underbrace{\frac{\gamma}{2} \int_0^T \psi_s^2 ds}_{\substack{\text{Market impact} \\ (\psi = \phi')}} + \underbrace{u(F_T - \phi_T)}_{\text{volume penalty}} \right].$$

Optimization in the presence of perfect forecast

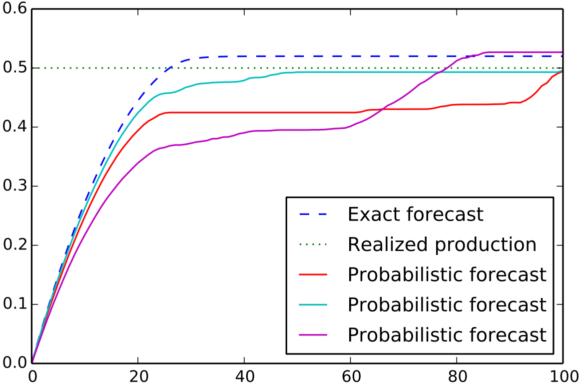
In the presence of exact forecast, the problem becomes deterministic:

$$\min_{\psi \geq 0} \left[\int_0^T \phi_t \mu_t dt + \frac{\gamma}{2} \int_0^T \psi_t^2 dt + u(F_T - \phi_T) \right].$$

In the examples below, we compare the deterministic solution for exact forecast with the stochastic solutions with random probabilistic forecasts.

The forecast trajectories are simulated so that all forecasts correspond to the same realized production

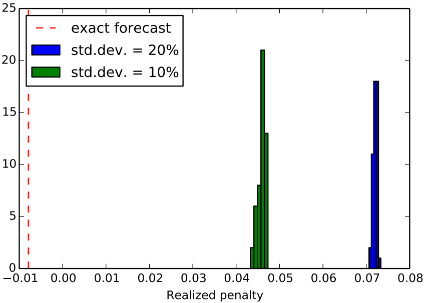
Examples

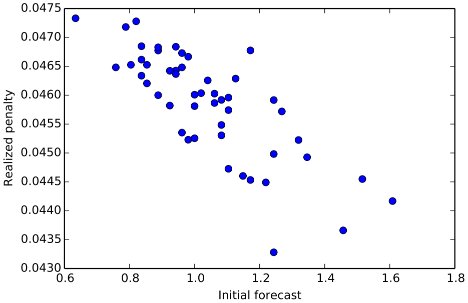


Sample selling strategies with market impact (but without price risk).

Strategies are updated dynamically as new information becomes available.

Examples





Left: realized penalty for different forecast quality.

Right: realized penalty as function of initial forecast.

Summary

- We use the truncated log-normal distribution to describe the wind power production;
- We propose a tractable dynamic model for the forecast errors parameterized by a volatility function which we estimate from the data;
- We express the gain of a power producer taking into account the volume risk, the price risk, and the production mismatch penalty;
- The optimal strategy is computed by solvinig numerically the HJB equation;
- We assess the value of probabilistic forecasts by comparing the realized gain with the case of exact forecast.

References

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